Numbers & Polys MAS3300

Exam-B

Prof. JLF King 31Aug2015

Note. This is an open brain, open (pristine) Sigmon-Notes exam. Please write each solution on a separate sheet of paper. Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

Every "**if**" must be matched by a "**then**."

B1: Using that $\mathbb{N}$ is WellOrdered, prove that there is no negative natnum. (<Thm2.3)

B2: YMAssume that $\mathbb{N}$ is sealed under addition. Please prove that $\mathbb{N}$ is sealed under multiplication. (<Thm2.7)

B3: Define a sequence $\vec{\mathbf{b}} = (b_0, b_1, b_2, \ldots)$ by $b_0 := 0$ and $b_1 := 3$ and

†: $$b_{n+2} := 7b_{n+1} - 10b_n, \quad \text{for } n = 0, 1, \ldots.$$

Use induction to prove, for all $k \in [0..\infty)$, that

‡: $$b_k = 5^k - 2^k.$$

Further. Given recurrence (†) and initial conditions, *explain* how you could have discovered/computed the numbers 5 and 2 in the (‡) formula.

Can you generalize to getting a (‡)-like formula when the recurrence is $\boxed{b_{n+2} := \mathbf{S}b_{n+1} - \mathbf{P}b_n}$, for arbitrary real-number coefficients **S** and **P**?

B4: Consider these sets:

$$B := \left\{-5 + \tfrac{n}{n+1} \mid n \in \mathbb{N}\right\};$$
$$C := \left\{7 - \tfrac{1}{n} \mid n \in \mathbb{Z}_+\right\};$$
$$D := \left\{n \cdot \sqrt{2} \mid n \in \mathbb{N}\right\};$$
$$\mathbb{Q}_{\geqslant 0} := \left\{\frac{p}{q} \mid p, q \in \mathbb{N} \text{ with } q \neq 0\right\}.$$

For each set below, circle one of "**WO**" "**Nope**". In the case of **Nope**, provide an explicit infinite decreasing sequence in the given set.

B	is	**WO**	**Nope**.
$C \cup D$	is	**WO**	**Nope**.
$\mathbb{Q}_{\geqslant 0}$	is	**WO**	**Nope**.
$\mathbb{Z} \setminus (-\infty..-17)$	is	**WO**	**Nope**.

End of Exam-B

B1: ___ ___ ___ 100pts

B2: ___ ___ ___ 100pts

B3: ___ ___ ___ 100pts

B4: ___ ___ 60pts

Total: ___ ___ ___ 360pts

Print name .... Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....

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