Numbers & Polys MAS3300 | **Exam-B** | Prof. JLF King 1Sep2015

Note. This is an open brain, open (pristine) SigmonNotes exam. Please write each solution on a separate sheet of paper. Write expressions unambiguously e.g, "$1/a + b$" should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with **negative** signs!)

B1: For free YMAssume that $\mathbb{N}$ is sealed under addition. Please prove that $\mathbb{N}$ is sealed under multiplication.

B2: Define a sequence $\vec{\mathbf{b}} = (b_0, b_1, b_2, \ldots)$ by $b_0 := 0$ and $b_1 := 3$ and

†: $$b_{n+2} := 7b_{n+1} - 10b_n\,, \quad \text{for } n = 0, 1, \ldots.$$

Use induction to prove, for all $k \in [0\,..\,\infty)$, that

‡: $$b_k = 5^k - 2^k\,.$$

Further. Given recurrence (†) and initial conditions, *explain* how you could have discovered/computed the numbers 5 and 2 in the (‡) formula.

Can you generalize to getting a (‡)-like formula when the recurrence is $\boxed{b_{n+2} := \mathbf{S}b_{n+1} - \mathbf{P}b_n}$, for arbitrary real-number coefficients $\mathbf{S}$ and $\mathbf{P}$? (This is Exer 2.15.2 in text.)

B3: Prove Thm3.3 (P.15), the Division Alg. Thm.

B4: Suppose $J \subset \mathbb{Z}$ is an ideal. Prove that there exists a natnum b so that each member of J is a multiple of b. (I am asking you to prove a little less than Thm3.6d (P.16). You'll want to use the Div.Alg.Thm.)

End of Exam-B

B1:	___ ___	60pts
B2:	___ ___	60pts
B3:	___ ___	60pts
B4:	___ ___	60pts
Total:	___ ___ ___	240pts

Print name .... Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....