Linear Algebra
MAS3114 3241

Class-B

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Hello. Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

Do ***not*** approx.: If your result is "$\sin(\sqrt{\boldsymbol{\pi}})$" then write that rather than $.9797\cdots$.

B1: Show no work.

z Prof. King wears bifocals, and cannot read small handwriting. Circle one: **True!** **Yes!** ***Who?***

a Consider these two matrices:

$$\mathsf{C} := \tfrac{1}{2}\cdot\begin{bmatrix}\sqrt{3} & -1\\ 1 & \sqrt{3}\end{bmatrix} \quad\text{and}\quad \mathsf{B} := \tfrac{1}{2}\cdot\begin{bmatrix}\sqrt{2} & \sqrt{2}\\ -\sqrt{2} & \sqrt{2}\end{bmatrix}.$$

Determine the matrix $[\mathsf{CB}]^{2007}=$ ____________ .

[*Hint:* You don't need multiply matrices. Geometrically, what motion do these matrices represent.]

b Suppose T is a linear map from $\mathbb{R}^3$ to $\mathbb{R}^2$. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for $\mathbb{R}^3$, and let $\{\mathbf{v}_1, \mathbf{v}_2\}$ be the standard basis for $\mathbb{R}^2$. Suppose that $T(\mathbf{e}_1) = 17\mathbf{v}_1 - 2\mathbf{v}_2$ and $T(\mathbf{e}_2) = 6\mathbf{v}_2$ and $T(\mathbf{e}_3) = -4\mathbf{v}_1 - 3\mathbf{v}_2$.

Then the matrix of T is: ____________ .

c Over $\mathbb{Q}$, the inverse of $\mathsf{E} := \begin{bmatrix}1 & x & z\\ & 1 & y\\ & & 1\end{bmatrix}$ is

.

d Let $\mathbf{v}_1 := \begin{bmatrix}0\\1\\-1\\0\end{bmatrix}$, $\mathbf{v}_2 := \begin{bmatrix}2\\1\\-2\\3\end{bmatrix}$, $\mathbf{v}_3 := \begin{bmatrix}4\\W\\3\\Y\end{bmatrix}$, So $\mathbf{v}_3 \in \mathrm{Spn}(\mathbf{v}_1, \mathbf{v}_2)$ when $W=$ ________ & $Y=$ ________ .

And $\mathbf{v}_3 = \alpha\mathbf{v}_1 + \beta\mathbf{v}_2$, where $\alpha=$ ________ and $\beta=$ ________ .

e The $3{\times}3$ elem-matrix whose lefthand action adds 8 times row-2 to row-1 is

.

f An example of $2{\times}2$-matrices with $\mathsf{A}^2 \neq \mathsf{B}^2$, yet with $\mathsf{A}^3 = \mathsf{B}^3$, is $\mathsf{A} =$ ________ and $\mathsf{B} =$ ________ .

g Each entry in $5{\times}5$ matrix M_x is a quadratic polynomial in x. So $\mathrm{Det}(\mathsf{M}_x)$ is a polynomial, and its maximum possible degree is ____________ .

Henceforth, let *AT* mean "Always True", *AF* mean "Always False" and *Nei* mean "<u>Nei</u>ther always true nor always false". Below, $\mathbf{u}, \mathbf{v}, \mathbf{w}$ repr. *distinct*, <u>*non-zero*</u> vectors in $\mathbb{R}^4$, a $\mathbb{R}$-VS. Please circle the correct response:

y1 If $\mathbf{w} \in \mathrm{Spn}\{\mathbf{u}, \mathbf{v}\}$ then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. *AT AF Nei*

y2 If $\mathbf{w} \notin \mathrm{Spn}\{\mathbf{u}, \mathbf{v}\}$ then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. *AT AF Nei*

y3 Collection $\{\mathbf{0}, \mathbf{w}\}$ is linearly-indep. *AT AF Nei*

y4 $\mathrm{Spn}\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{u} + 2\mathbf{v} + 3\mathbf{w}\}$ is all of $\mathbb{R}^4$. *AT AF Nei*

y5 If none of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ is a multiple of the other vectors, then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. *AT AF Nei*

y6 For $2{\times}2$ matrices: $\mathrm{Det}(\mathsf{B} + \mathsf{A}) = \mathrm{Det}(\mathsf{B}) + \mathrm{Det}(\mathsf{A})$. *AT AF Nei*

y7 For $2{\times}2$: $\mathrm{Det}([\mathsf{BA}]^{2008}) = \mathrm{Det}(\mathsf{B}^{2008}) \cdot \mathrm{Det}(\mathsf{A}^{2008})$. *AT AF Nei*

Essay question: On your own sheets of paper, write a soln using complete sentences, explaining a bit about HOW this problem is solved.

B2: A system of 3 linear equations in unknowns $x_1, \ldots, x_5$ reduces to the augmented matrix

$$\left[\begin{array}{ccccc|c}1 & 1 & 0 & 0 & 1 & 12\\ 0 & 0 & 1 & 0 & -8 & 34\\ 0 & 0 & 0 & 1 & 5 & -56\end{array}\right],$$

which is *almost* in RREF. Please circle each *pivot*.

OYOP, describe the *general solution* in this form,

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix} = \begin{bmatrix}?\\?\\?\\?\\?\end{bmatrix} + \alpha\begin{bmatrix}?\\?\\?\\?\\?\end{bmatrix} + \beta\begin{bmatrix}?\\?\\?\\?\\?\end{bmatrix} + \gamma\begin{bmatrix}?\\?\\?\\?\\?\end{bmatrix} + \ldots$$

where each $\alpha, \beta, \gamma, \delta, \ldots$ is a free variable (either x_1 or…or x_5), and each column vector has specific numbers in it. $\mathrm{Dim}(\mathit{SolnFlat}) =$ ____________ .

B3: State the Cramer's Rule Thm.

Apply C.R. to give a formula for $x_2 =$ ____________

ITOf A, B, C, D, t_1, t_2, where $\begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}t_1\\t_2\end{bmatrix}$.

End of Class-B

B1:	___ ___ ___	200pts
B2:	___ ___	45pts
B3:	___ ___	45pts
Total:	___ ___ ___	290pts

Print name .... Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....