5 Mar. 1998 Linear Algebra 3114 — In-class B — Prof. J. King

Note: I use IFF for "if and only if".

B1: SHORT ANSWER. Fill in the blanks below, expressing your answer in simplest form unless otherwise indicated. Write **DNE**, for "Does Not Exist", if the indicated operation cannot be performed.

Do not make approximations. **Show no work.** There is no partial credit for this question, so carefully verify that you have written what *you* mean. In particular, make sure that you write expressions unambiguously, e.g the expression "$1/a+b$" should be parenthesized either $(1/a)+b$ or $1/(a+b)$ so that I know your meaning. Be careful with negative signs. [Points $30 * 9 = 270$]

(a) Write the uppercase versions of the following Greek letters, along with their names.
Example: "α: ______." You fill in: A (alpha).

η: ______ λ: ______ σ: ______ μ: ______ γ: ______

(b) Let $F := \begin{bmatrix} 2 & 0 & -4 \\ 1 & 1 & 1 \\ 3 & -1 & -9 \end{bmatrix}$. Determine its reduced row-echelon form. $\operatorname{rref}(F) =$ ______.

(c) The 2×2 matrix $A := \begin{bmatrix} x & 5 \\ y & z \end{bmatrix}$ is non-singular IFF ______, and then $A^{-1} =$ ______.

(d) Compute the inverse of $F := \begin{bmatrix} 1 & -2 & 3 & 5 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$.

$F^{-1} =$ ______.

(e1) Let $D := \begin{bmatrix} 3 & \\ & 1 \end{bmatrix}$. Then $D^{100} =$ ______.

(e2) Let $A := \begin{bmatrix} 7 & -4 \\ 6 & -3 \end{bmatrix}$. Observe that $A = PDP^{-1}$, where $P := \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$. Compute $A^{100} =$ ______.

(f) Give an example of two 2×2 matrices $A \neq B$ for which $A^2 = B^2$. $A =$ ______ and $B =$ ______.

(g) Suppose A is an $n \times n$ matrix in ref (*not* rref!), and $\mathbf{v}$ is some $n \times 1$ column-vector. In the worst case, what is the number of multiplications needed to compute a solution to $A\mathbf{x} = \mathbf{v}$? Number= ______.

(h) Suppose T is a linear map from $\mathbb{R}^3$ to $\mathbb{R}^2$. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for $\mathbb{R}^3$, and let $\{\mathbf{v}_1, \mathbf{v}_2\}$ be the standard basis for $\mathbb{R}^2$. Suppose $T(\mathbf{e}_1) = 2\mathbf{v}_1 - \mathbf{v}_2$ and $T(\mathbf{e}_2) = 4\mathbf{v}_2$ and $T(\mathbf{e}_3) = 3\mathbf{v}_1 + 5\mathbf{v}_2$.

Then the matrix of T is: ______.

B2: [50 Points] Suppose $\mathbb{V}$ and $\mathbb{W}$ are vectorspaces, and $T\colon \mathbb{V} \to \mathbb{W}$. Give a precise definition (on a separate sheet) of what it means for T to be ***linear***.

Give an example of a map $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ which is **not** linear. (Naturally, you need to show that your T is not linear. Give specific vectors and state which linearity property they fail.)

B3: [80 Points] Let R_θ denote the 2×2 matrix whose action is to rotate the plane counterclockwise by θ (radians, natch). Show me R_θ.

Derive the formulae for $\cos(\alpha+\beta)$ and $\sin(\alpha+\beta)$ as we did in class, *explaining* what you are doing and *why* it works.

B4: [70 Points] Consider these two matrices:

$$R := \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \qquad \text{and} \qquad A := \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}.$$

Determine the matrix $(RA)^{36}$ via any method, and explain your method.R_θ
([Hint: You don't need multiply matrices. Think of the linear transformations that these matrices represent.])

Extra Credit: **EC1** [10 Points] Give an example of two 2×2 matrices $C \neq D$ for which $C^3 = D^3$. $C =$ ⌞....................⌟ and $D =$ ⌞....................⌟.

EC2 [10 Points] Make up a nice Linear Algebra problem –something creative. You do not necessarily need to know how to solve it.

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