Linear Algebra MAS4105 *6137* — **Class-B** — Prof. JLF King, Tuesday, 27Oct2015

B1: Show no work. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a) Perm $\pi := [4,5,6,7,8,1,2,3]$ has $\mathrm{Sgn}(\pi) =$ **+1** **−1**.

b) Every 5×5 matrix M has $\mathrm{Det}(7\mathsf{M}) = \alpha\cdot\mathrm{Det}(\mathsf{M})$, where α is [circle]

7^{25} 7^5 7^4 7 5^7 25^7 5^{49} None-of-these

c) Inverse of $\begin{bmatrix}1 & & \\ 4 & -1 & \\ 5 & 6 & 2\end{bmatrix}$ is $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$.

d) Determinant of $\mathsf{M} := \begin{bmatrix}-5 & 4 & 0\\ 1 & 3 & -2\\ 2 & 1 & 0\end{bmatrix}$ is ______.
The characteristic-poly of M is $Ax^3 + Bx^2 + Cx + D$, where $B=$ ______ and $C=$ ______.

e) Mats $\mathsf{U}=\begin{bmatrix} & \\ & \end{bmatrix}$, $\mathsf{V}=\begin{bmatrix} & \\ & \end{bmatrix}$

are $\mathbb{R}$-matrices such that $\mathsf{U}^2 \neq \mathsf{V}^2$, yet $\mathsf{U}^3 = \mathsf{V}^3$.

f) Let $\mathsf{R}(x) := \begin{bmatrix}3x^2 & 7x^5 - 8 & 6x\\ 5 & 9x & 2x-1\\ 8x+4 & x^6+1 & x^4+2\end{bmatrix}$.
The high-order term of polynomial $\mathrm{Det}(\mathsf{R}(x))$ is Cx^N, where $C =$ ______ and $N =$ ______.

g) Let $\mathsf{T}:\mathbb{R}^2\to\mathbb{R}^2$ by reflecting the plane across the θ-angle line. [Picture on blackboard.] Then $\mathsf{M} := [\![\mathsf{T}]\!]_{\mathcal{E}}^{\mathcal{E}}$ equals $\begin{bmatrix}\cos(\alpha) & \cos(\beta)\\ \sin(\alpha) & \sin(\beta)\end{bmatrix}$, where $\alpha =$ ______ and $\beta =$ ______. Also, $\mathrm{Det}(\mathsf{M}) =$ ______.

OYOP: Essay: *Write on every **third** line, so that I can easily write between the lines. In grammatical English **sentences**, prove the following:*

B2: Here, each matrix is a 2×2 matrix. Write $\mathsf{C} \looparrowright \mathsf{D}$, "matrix C ***loop-arrows*** D", if *there exists* an invertible U such that $\mathsf{UCU}^{-1} = \mathsf{D}$.

x) What are the properties that that $\looparrowright$ needs to have, to be an *"equivalence relation"*?
Prove that $\looparrowright$ is an equivalence relation.

y) Prove: *If at least one of* S,T *is invertible then* $\mathsf{ST} \looparrowright \mathsf{TS}$.

z) Matrices $\mathsf{A} = \begin{bmatrix} & \\ & \end{bmatrix}$, $\mathsf{B} = \begin{bmatrix} & \\ & \end{bmatrix}$

are such that $\mathsf{AB} \not\looparrowright \mathsf{BA}$, *with proof*.

End of Class-B

B1:	___ ___ ___	160pts
B2:	___ ___	80pts
Total:	___ ___ ___	240pts