Linear Algebra MAS4105 *6137*	**Class-B**	Prof. JLF King *Touch*: 2Oct2015

B1: OYOP: Essay: *Write on every* ***third*** *line, so that I can easily write between the lines. In grammatical English* ***sentences****, prove the following:*

Intersection thm. *Suppose* **X** *and* **Y** *are subspaces of VS* **V**. *Prove that* $\mathbf{W} := \mathbf{X} \cap \mathbf{Y}$ *is a subspace.*

[*Hint:* First give a formal *definition* of what it means for a subset of **V** to be a sub*space*.]

B2: Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

___ ___ **a** With C the change-of-basis matrix from $\mathcal{E} := (1, x, x^2)$ to $\mathcal{B} := (3x + 5x^2, x + 2x^2, 1)$, then C^{-1} equals

$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$, C= $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$.

___ ___ **b** Over $\mathbb{Q}$, the inverse of $E := \begin{bmatrix} 1 & A & C \\ & 1 & B \\ & & 1 \end{bmatrix}$ is

$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$.

___ ___ **c** Glued to a massless plate is a 15 lb weight at the origin, a 5 lb weight at the point $(3, -1)$, and 10 lb at point (______ , ______), thus putting the center-of-mass of the weighted-plate at $(1, 2)$.

___ ___ **d** Let $M := \begin{bmatrix} 1 & 5 & -1 & -20 & -32 \\ 0 & 2 & 5 & -1 & 33 \\ 0 & 1 & 3 & 0 & 21 \end{bmatrix}$. Working over field $\mathbb{Z}_7$, matrix *RREF*(M) equals (write entries in [-3 .. 3])

$\begin{bmatrix} & & & & \\ & & & & \\ & & & & \end{bmatrix}$

___ ___ **e** Let R_θ be the std. rotation [by θ] matrix. With

$$C := \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix} \quad \text{and} \quad B := \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix},$$

the product $[CB]^{35} = \alpha \cdot R_\theta$, with $\alpha=$ ______ $\in\mathbb{R}$ and $\theta=$ ______ $\in (-180°, 180°]$. [*Hint:* Don't multiply matrices. Geometrically, C and B represent what lin-trns?]

B3: *Henceforth*, let **AT** mean "Always True", **AF** mean "Always False" and **Nei** mean "NEIther always true nor always false". Vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{Q}^4$ [a $\mathbb{Q}$-VS], and none is $\vec{\mathbf{0}}$. Here, use {} for a multi-set.

z1 If none of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ is a multiple of the other vectors, then $S := \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. AT AF Nei

z2 If $S := \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is lin-indep, then so is $\{2\mathbf{u}, 3\mathbf{v}, 4\mathbf{w}\}$. AT AF Nei

z3 $\text{Spn}\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{u} + 2\mathbf{v} + 3\mathbf{w}\}$ is all of $\mathbb{Q}^4$. AT AF Nei

z4 If $\mathbf{w} \notin \text{Spn}\{\mathbf{u}, \mathbf{v}\}$ *and* $\mathbf{u} \notin \text{Spn}\{\mathbf{v}, \mathbf{w}\}$, then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. AT AF Nei

z5 Collection $\{\mathbf{0}, \mathbf{w}\}$ is linearly-dependent. AT AF Nei

y7 For 2×2 matrices: $\text{Det}(B + A) = \text{Det}(B) + \text{Det}(A)$. AT AF Nei

y8 For 2×2: $\text{Det}([BA]^{2008}) = \text{Det}(B^{2008}) \cdot \text{Det}(A^{2008})$. AT AF Nei

End of Class-B

B1:	___ ___	45pts
B2:	___ ___ ___	150pts
B3:	___ ___	50pts
Total:	___ ___ ___	245pts