Linear Algebra MAS4105 *3247* | **Class-B** | Prof. JLF King 31Aug2015

B1: A system of 3 linear equations in unknowns $x_1, \ldots, x_5$ reduces to the augmented matrix

$$\begin{bmatrix} 5 & 4 & 0 & 0 & 10 & | & -15 \\ 0 & 0 & 3 & 0 & -8 & | & -3 \\ 0 & 0 & 0 & 2 & 1 & | & 0 \end{bmatrix},$$

which is *almost* in RREF. Please circle each *pivot*.

OYOP, describe the *general solution* in this form,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \alpha \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \beta \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \gamma \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \ldots$$

where each $\alpha, \beta, \gamma, \delta, \ldots$ is a free variable (either x_1 or...or x_5), and each column vector has specific numbers in it. Dim(*SolnFlat*) = ________________.

B2: Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

___ ___ **a** $\mathsf{M} := \begin{bmatrix} 2 & 3 \\ 11 & 4 \end{bmatrix}$. Compute M^{-1} over these three fields.

Over $\mathbb{Z}_5$: M^{-1}= ________.

Over $\mathbb{Z}_7$: M^{-1}= ________. Over $\mathbb{Q}$: M^{-1}= ________.

___ ___ **b** Over $\mathbb{Q}$, the inverse of $\mathsf{E} := \begin{bmatrix} 1 & x & z \\ & 2 & y \\ & & 3 \end{bmatrix}$ is

___ ___ **c** Glued to a massless plate is a 15 lb weight at the origin, a 5 lb weight at the point $(3, -1)$, and 10 lb at point (________ , ________), thus putting the center-of-mass of the weighted-plate at $(1, 2)$.

___ ___ **d** Let $\mathsf{M} := \begin{bmatrix} 1 & 5 & -1 & -20 & -32 \\ 0 & 2 & 5 & -1 & 33 \\ 0 & 1 & 3 & 0 & 21 \end{bmatrix}$. Working over field $\mathbb{Z}_7$, matrix *RREF*(M) equals (write entries in [-3 .. 3])

e Since $M := 211$ is prime, ring $\mathbb{Z}_{211}$ is a field. So the mod-M reciprocal of 199 is $R :=$ ________ $\in [0 \,..\, M)$.

[IOWords, $199 \cdot R \equiv_{211} 1$ and $R \in [0 \,..\, M)$.]

B3: *Henceforth*, let **AT** mean "Always True", **AF** mean "Always False" and **Nei** mean "NEIther always true nor always false". Vectors $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{Q}^4$ [a $\mathbb{Q}$-VS], and none is $\vec{\mathbf{0}}$. Here, use $\{\}$ for a multi-set.

z1 If none of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ is a multiple of the other vectors, then $S := \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. AT AF Nei

z2 If $S := \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is lin-indep, then so is $\{2\mathbf{u}, 3\mathbf{v}, 4\mathbf{w}\}$. AT AF Nei

z3 $\text{Spn}\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{u} + 2\mathbf{v} + 3\mathbf{w}\}$ is all of $\mathbb{Q}^4$. AT AF Nei

z4 If $\mathbf{w} \notin \text{Spn}\{\mathbf{u}, \mathbf{v}\}$ *and* $\mathbf{u} \notin \text{Spn}\{\mathbf{v}, \mathbf{w}\}$, then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. AT AF Nei

z5 Collection $\{\mathbf{0}, \mathbf{w}\}$ is linearly-dependent. AT AF Nei

y7 For 2×2 matrices: $\text{Det}(\mathsf{B} + \mathsf{A}) = \text{Det}(\mathsf{B}) + \text{Det}(\mathsf{A})$. AT AF Nei

y8 For 2×2: $\text{Det}([\mathsf{BA}]^{2008}) = \text{Det}(\mathsf{B}^{2008}) \cdot \text{Det}(\mathsf{A}^{2008})$. AT AF Nei

End of Class-B

B1:	___ ___	55pts
B2:	___ ___ ___	150pts
B3:	___ ___	50pts
Total:	___ ___ ___	255pts

Please PRINT your name and ordinal. Ta:

________________________________ Ord:

Honor Code: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________________________________