Linear Algebra MAS4105 5441 | **B-InClass** | Prof. JLF King 31Aug2015

**B6:** Short answer: Show no work. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

z: $\mathsf{M}^\star$ means: Circle: **Adjoint of M** ***Keanu Reeves***

a: Multinomial coeff $\binom{8}{3,3,4,2}$= ________.

In $[x+y]^{12}$, the coeff of $x^3y^9$ is ________.

b+: Let $\mathsf{B} := \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$. [*Hint:* Write B as a diag-matrix plus a nilpotent matrix.]

Then $\mathsf{B}^{12}$ = ________.

c+: $\mathsf{G} := \left[\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right]$. Produce a diagonal-matrix

D= ________ and non-sing Q= ________, possibly *complex*, so that $\mathsf{QDQ}^{-1} = \mathsf{G}$.

d: $\mathsf{C} := \begin{bmatrix} 2 & 0 & -5 \\ -1 & -1 & 2 \\ 3 & 0 & 1 \end{bmatrix}$. Matrix $P_{\mathsf{C}}(-1)$= ________.

Char-poly $\wp_{\mathsf{C}}(x)$= ________.

e: In each blank below, write either "there exist" or "for all" and Circle one of each underlined scalar-pairs. The phrase $\boxed{\mathrm{Spn}(\mathbf{v},\mathbf{w}) \supset \mathrm{Spn}(\mathbf{x},\mathbf{y})}$ means:

" ________ *scalars* $\underline{a,b}$ | $\underline{c,d}$ (*st.* | *we have* | *and*)

________ *scalars* $\underline{a,b}$ | $\underline{c,d}$ (*st.* | *we have that*)

$\boxed{a\mathbf{v} + b\mathbf{w} = c\mathbf{x} + d\mathbf{y}}$."

f: Give 2×2 M= ________, col-vec $\mathbf{v}$= ________, st. $\mathbf{v}$ is not an M-evec, but is an $\mathsf{M}^2$-evec with $\mathsf{M}^2$-eval= ________. Everything is over $\mathbb{Q}$.

*Below, let AT mean "Always True", AF mean "Always False" and Nei mean "Neither always true nor always false".* Below, M is a square matrix over $\mathbb{Q}$.

g: If $\wp_{\mathsf{M}}()$ splits over $\mathbb{Q}$, then M is $\mathbb{Q}$-diagonalizable.
*AT AF Nei*

**B7:** State and prove Cramer's Thm. Let $\mathsf{E} := \begin{bmatrix} x & y & -1 \\ 7 & 1 & z \\ 1 & 1 & 3 \end{bmatrix}$. Let $h(x,y,z)$ be the $(3,1)$-entry of $\mathsf{E}^{-1}$. Then $h(x,y,z)$= ________, a ratfnc.

**B8:** Matrix $\mathsf{M} = \left[\begin{smallmatrix} \mathsf{A} & \mathsf{B} \\ \mathsf{C} & \mathsf{D} \end{smallmatrix}\right]$, where A and D are 5×5 and 7×7, resp. Suppose C is the 7×5 zero-matrix. Prove that $\mathrm{Det}(\mathsf{M}) = \mathrm{Det}(\mathsf{A}) \cdot \mathrm{Det}(\mathsf{D})$. [*Hint:* A good picture helps.]

**B9:** *OASSOP, write out the following sentences, and complete them to give the correct definitions. Be specific with phrases "every", "some", "there exists", etc.. Define "trivial soln" before using it. Let* $\mathbf{V} := \mathbb{C}^{\times 5}$. *All matrices below are* 5×5 *complex matrices UOS.* (*Unless Otherwise Stated.*)

*Collection* $\mathcal{C} := \{\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_K\}$ *of* $\mathbf{V}$*-subspaces is **linearly independent** IFF ...*

*Matrix B is **doubly stochastic** if ...*

Fix $\beta \in \mathbb{C}$. *The* M-***algebraic-multiplicity*** *of* $\beta$ *is .... The* M-***geometric-multiplicity*** *of* $\beta$ *is ....*

*Degree-5 monic poly* $g(x)$ ***splits*** *over* $\mathbf{F}$ *IFF ...*

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|---|---|---|
| **B-Home:** | ___ ___ ___ | 540pts |
| **B6:** | ___ ___ ___ | 130pts |
| **B7:** | ___ ___ | 60pts |
| **B8:** | ___ ___ | 60pts |
| **B9:** | ___ ___ | 55pts |
| **Total:** | ___ ___ ___ | 845pts |

*Please PRINT your name and ordinal. Ta:*

________ Ord: ________

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________

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