Differential Eqns MAP2302 **B-Class** Prof. JLF King 2Sep2015

***Hello.*** Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. **Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with negative signs!)**

Do ***not*** approx.: If your result is "$\sin(\sqrt{\boldsymbol{\pi}})$" then write that rather than $.9797\cdots$.

**Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.**

Write *rational numbers* as fractions: E.g $\frac{1}{2}$ and $1/3$, but not 0.5 nor 0.3333...; **use fractions**.

**B1:** Show no work.

a) Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

**True!** **Yes!** ***wH'at S a?sEnTENcE***

b) A tank initially holds 80gal of $2\frac{\text{lb}}{\text{gal}}$ brine. Pipe-1 feeds the tank, at rate $3\frac{\text{gal}}{\text{min}}$, with brine of time-varying salinity $t^3\,\frac{\text{lb}}{\text{gal}}$. Pipe-2 feeds the tank at $2\frac{\text{gal}}{\text{min}}$, brine of salinity $\mathsf{e}^t\,\frac{\text{lb}}{\text{gal}}$. The tank discharges brine at rate $9\frac{\text{gal}}{\text{min}}$. Until the tank empties, the tank holds $W(t) = [\quad\quad]$gal; it empties in ______ min.

Finally, $y(t)$, the number of pounds of salt in the tank at time $t$, satisfies FOLDE $\frac{\mathrm{d}y}{\mathrm{d}t} + F(t)\cdot y = H(t)$, where $F(t)=$ ______

and $H(t)=$ ______.

c) DE $\left[\mathcal{N}(x,y)\cdot\frac{\mathrm{d}y}{\mathrm{d}x}\right] + \mathcal{M}(x,y) = 0$ is *exact*, where

$$\mathcal{N}(x,y) := \left[7+\mathsf{e}^x\right] \quad\text{and}\quad \mathcal{M}(x,y) := \mathsf{e}^x\left[y-2x\right].$$

Its soln $y = y(x)$ satisfies $\mathbf{F}\big(x, y(x)\big) = \text{Const}$, where $\mathbf{F}\big(x,y\big) =$ ______.

d) DE $\left[[x^2 - xy]\cdot\frac{\mathrm{d}y}{\mathrm{d}x}\right] + xy - 1 = 0$ is not, alas, *exact*. Happily, multiplying both sides by (non-constant) fnc

$W(x) =$ ______

gives a *new* DE which *is* exact. **Did you** *Check?*

e $\underline{A}$ soln to $[f'' - 3f'](x) = \text{-}[6x+1]$ is **polynomial** $f(x)=$ ________________. Using parameters $\alpha$ and $\beta$, then, the *general* solution to $[h'' - 3h'](x) = \text{-}[6x+1]$ is

$h_{\alpha,\beta}(x)=$ ____________________________________.

And $\underline{\text{the}}$ $h$ with $h(0) = 2$ and $h'(0) = 7$ is $h(x)=$ ________________________________________.

f Matrix-product $\begin{bmatrix} 4 & 7 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 3 & \text{-}1 \\ 2 & 1 \end{bmatrix} =$ ______________.

With $\mathsf{A} := \begin{bmatrix} 4 & 7 \\ 2 & 4 \end{bmatrix}$, then the $(2, 1)$-entry of $\mathsf{A}^{-1}$ is ______________.

End of B-Class

**B1:** ___ ___ ___ 185pts

**Total:** ___ ___ ___ 185pts

*Please PRINT your* name and ordinal. *Ta:*

______________________________________________ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______________________________________