NT Summer
MAS4203 4801

Class-B

Prof. JLF King
Monday, 25Jul2016

Please *fill-in* every *blank* on this sheet. [50 minute exam, *or so Prof. Erroneous thought*.]

B1: *Show no work.*

a Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

True! **Yes!** ***wH'at S a?sEnTENcE***

b Define the ***numeral map*** $h{:}[1..12]\circlearrowleft$, where $h(n)$ is the number of letters in the n^{th} numeral. So $h(12)$ equals 6, since "twelve" has 6 letters.
Compute the convolution $[h \circledast \boldsymbol{\mu}](10)=$ ______ .

c Consider the four congruences C1: $z \equiv_8 1$, C2: $z \equiv_{18} 15$, C3: $z \equiv_{21} 18$ and C4: $z \equiv_{10} 3$. Let z_j be the *smallest natnum* satisfying (C1) $\overset{\text{All}}{\therefore}$ (Cj). Then
$z_2=$ ______ ; $z_3=$ ______ ; $z_4=$ ______ .

d Let $N := 5^8$. Then $x^2 + y^2 = N$, where posints $x < y$ and $x \perp y$. [*Hint:* Use "repeated SOTS-melding". Only three melds needed.]
$x =$ ______ and $y =$ ______ .

e TMWFIt, 8 is a mod-125 primroot, since its mult-order (mod 125) is 100 $\stackrel{\text{note}}{=\!=}$ $\varphi(125)$. Use the CRT-isomorphism to compute <u>the</u> corresponding mod-250 primroot $R =$ ______ $\in [0..250)$.

f $S(98,000,000)=$ ______ where, for posints k, let $S(k)$ be the number of mod-k square-roots of 1. BTWay, group $(\Phi(1024), \cdot, 1)$ is isomorphic to this product ______ of cyclic groups.
[Let $\mathbf{C}(N)$ denote the cyclic group with N many elements.]

OYOP: *In grammatical English* ***sentences****, write your essays on every* ***third*** *line* (usually)*, so that I can easily write between the lines. Start each essay on a* ***new*** *sheet of paper.*

B2: Note $f(n) := \frac{1}{2}\cdot[27^n + 31^n]$ is an integer. Prove, for each <u>*odd*</u> $n \geqslant 5$, that $f(n)$ is composite. [*Hint:* Look at $f(n)$ mod something.]

B3: For prime p, the units group $\Gamma := \Phi_p$ is cyclic of order $p-1$. Let $\mathcal{S}$ be its set of generators [those elts of order $p-1$]. For $p>3$, prove $\prod(\mathcal{S}) \equiv_p 1$. [Use Wilson's-Thm ideas.]

B1:	___ ___ ___	120pts
B2:	___ ___	45pts
B3:	___ ___	40pts
Total:	___ ___ ___	205pts