Staple!

Combinatorics 1 MAD4203 | **Class-B** | Prof. JLF King *Touch*: 22Oct2018

B1: Show no work.
Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a The physics lab has atomic *zinc*, *tin*, *silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] ______ many possibilities.

b Equality $\binom{60}{17,21,22} = \binom{60}{21} \cdot \binom{N}{K}$ suggests that $N =$ ______ and $K =$ ______.

c The coeff of $x^7 y^{12}$ in $[5x + y^3 + 1]^{30}$ is ______.

[You may write in form number times multinomial-coeff. You can leave the multinomial-coeff as such, or write ITOf factorials.]

d Given sets with cardinalities $|B| = 8$ and $|E| = 5$, the number of non-constant fncs in B^E is ______.

e Mimicking what we did in class: From the 987×200 game-board, cut-out (remove) the **(**35, 150**)**-cell and one other cell at $P = (x, y)$. Circle those choices for P,

(150, 160**)**, **(**14, 35**)**, **(**66, 77**)**, **(**195, 15**)**, **(**123, 4**)**

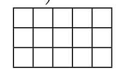
which, if removed, would leave a board that *definitely* can**not** be domino-tiled.

OYOP: *In grammatical English* ***sentences****, write your essays on every* ***third*** *line* (usually)*, so that I can easily write between the lines.*

B2: Give a careful bijective proof of:

Thm: *Fix a natnum* $N \geqslant 2$*. Then*

$$*: \quad [N^2 - N] \cdot 2^{N-2} = \sum_{k=2}^{N} [k^2 - k] \cdot \binom{N}{k}.$$

B3: An ***Lmino*** (pron. "ell-mino") comprises three  squares in an "L" shape (all four orientations are allowed). For natnum N, let $\mathbf{R}_N$ denote the $3{\times}N$ board: I.e,  is the $\mathbf{R}_5$ board. Prove:

Theorem: When N is odd, then board $\mathbf{R}_N$ is *not* *Lmino-tilable.*

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, *labeled* pictures.

End of Class-B

B1:	___ ___ ___	125pts
B2:	___ ___	45pts
B3:	___ ___	75pts
Total:	___ ___ ___	245pts

Please PRINT your name and ordinal. Ta:

Ord: ______

Honor Code: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______