Staple!

Ord: __ __

Algebra 1 MAS4301 3175 **In-class-B** Prof. JLF King Tuesday 08Apr2003

Note. Open brain, closed book/notes. Use $\varphi()$ for the Euler phi-fnc. Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$. Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!) Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than $.9797\cdots$. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

B7: Two-second-teasers: Show no work. (Pts: 20×2, 30×3.)

__ __ z Since Prof. King has cycle-lengths 2, 2, 3, he is obviously *Circle*: **Even**-handed. Just-plain-**Odd**.

__ __ a $\text{Aut}(\mathbb{D}_{15})$ has ________ many elements? $\text{Inn}(\mathbb{D}_{15})$ has ________ many elements?

__ __ b The *cyclic* group $\mathbf{U}(125)$ has ________ many elements and ________ many *generators*? (Express each ans. as a product.)

__ __ b Integers $A =$ ________ and $B =$ ________ are such that

$$(y, z) \mapsto [Ay + Bz]_{\text{mod } 104}$$

is a ring-iso from $\mathbb{Z}_8 \times \mathbb{Z}_{13} \to \mathbb{Z}_{104}$. Give an explicit integer $n=$ ________ st. $n \equiv_8 3$ and $n \equiv_{13} 1$.

__ __ c Consider poly $f(x) := [x-4][x-13]$. How many *distinct* roots does f have mod 9?: ________ Mod 2^{100}?: ________. Mod 170?: ________.

__ __ d If the peg-jump pattern on the BB can jump down to a single peg, then the Klein-4 argument shows that the only possible last-peg positions are ________ and ________. (E.g H 12 or J 92.)

Essay questions. Please write (on your own paper) in complete grammatical sentences.

B8: For an arb. gp G, prove: $\text{Inn}(G) \triangleleft \text{Aut}(G)$.

B9: For Γ a comm.ring, define terms ***maximal ideal*** and ***prime ideal***. For $J \subset \Gamma$ an ideal, prove that

$$J \text{ a maximal ideal} \implies \Gamma/J \text{ is a field}.$$

Give, with proof, an explicit comm.ring Γ and *prime ideal* J which is not a maximal ideal.

End of In-class-B

Hm-B:	__ __ __	455pts
B7:	__ __ __	130pts
B8:	__ __	35pts
B9:	__ __	65pts
Total:	__ __ __	685pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

____ Ord: __ __