Combinatorics
MAD4203 3214

Class-B

Prof. JLF King
Wednesday, 18Oct2017

Welcome. Write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

Use $\boldsymbol{\mathcal{S}}(N, K)$ for 2Stirling #s, and use $\mathbf{c}(N, K)$ for the signless-1Stirling #s.

B4: Short answer. Show no work.

a Which is *optional*? circle *Writing-in-<u>sentences</u>.* *Writing-t-<u>different</u>-from-+.* *Writing-LARGE.* *Um...*

b The Bell-number recurrence relation we discussed in class is

$$\forall K \in \mathbb{N}: \ B(K+1) = \sum_{n=\ell}^{K} [\mu_n \cdot B(n)], \text{ where}$$

$\ell =$ 0 and $\mu_n = \binom{K}{n}$.

[N.B: The μ_n numbers may depend on K.]

c For $N \geqslant K \geqslant 1$, we have recurrence relations:

$\boldsymbol{\mathcal{S}}(N, K) =$ $\boldsymbol{\mathcal{S}}(N-1, K-1) + K \cdot \boldsymbol{\mathcal{S}}(N-1, K)$.

$\mathbf{c}(N, K) =$ $\mathbf{c}(N-1, K-1) + [N-1] \cdot \mathbf{c}(N-1, K)$.

d $\mathbf{c}(L, L-1) = \binom{L}{2} = \frac{L[L-1]}{2}$ [Closed formula].

Fnc $f(n) := \mathbf{c}(n, n-2)$ is a polynomial of degree 4.

e Define G:$[1..12]\circlearrowleft$ where $G(n)$ is the number of letters in the n^{th} Gregorian month. So $G(2) = 8$, since the 2^{nd} month is "February". The only fixed-point of G is 9. The *set* of posints k where $G^{\circ k}(12) = G^{\circ k}(7)$ is $[4..\infty)$.

[January, February, March, April, May, June, July, August, September, October, November, December]

Soln: **Month-map questions.** Here is the orbit-structure of G:

```
           3       7 <---- 1
T                    \                /-----<  9
w C        ^        |  \-- 10        |
o y        |        |                | One-    ^
  c        |        |                |  cycle  |
  l        v        v                |         |
  e                        G-Map      \_______/
           5 <---- 4

                    ^           /--- 2
                    |          /
                    6 <----- 8 <---- 11
                               \
                                \-- 12
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OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually)*, so that I can easily write between the lines.* Start each essay on a <u>new</u> sheet-of-paper. Please number the pages "1 of 57", "2 of 57"... (or "1/57", "2/57"...) I suggest you put your name on each sheet.

B5: i Give our formal definition of what it means for a $\pi \in \mathbb{S}_N$ to be an ***even*** [i.e, +1] or an ***odd*** permutation [i.e, -1]. This is call the ***sign*** of π, written, $\operatorname{Sgn}(\pi)$.

ii Prove, $\forall \beta, \alpha \in \mathbb{S}_N$, that $\operatorname{Sgn}(\beta \circ \alpha) = \operatorname{Sgn}(\beta) \cdot \operatorname{Sgn}(\alpha)$.

End of Class-B

B4:	___ ___	95pts
B5:	___ ___	55pts
Total:	___ ___ ___	150pts