

**Welcome.** Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Use  $\mathcal{S}(N, K)$  for 2Stirling #s, and use  $\mathbf{c}(N, K)$  for the signless-1Stirling #s.

**B4:** Short answer. Show no work.

**a** Which is *optional*? circle *Writing-in-sentences.*  
*Writing-t-different-from-+.* *Writing-LARGE.* Um...

**b** The Bell-number recurrence relation we discussed in class is

$$\forall K \in \mathbb{N}: B(K+1) = \sum_{n=0}^K [\mu_n \cdot B(n)], \text{ where}$$

$\ell = 0$  and  $\mu_n = \binom{K}{n}$ .  
[N.B: The  $\mu_n$  numbers may depend on  $K$ .]

**c** For  $N \geq K \geq 1$ , we have recurrence relations:

$$\mathcal{S}(N, K) = \mathcal{S}(N-1, K-1) + K \cdot \mathcal{S}(N-1, K)$$

$$\mathbf{c}(N, K) = \mathbf{c}(N-1, K-1) + [N-1] \cdot \mathbf{c}(N-1, K)$$

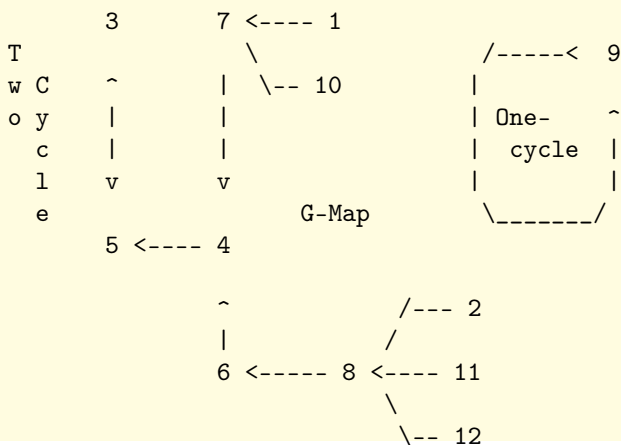
**d**  $\mathbf{c}(L, L-1) = \binom{L}{2} = \frac{L[L-1]}{2}$  [Closed formula].

Fnc  $f(n) := \mathbf{c}(n, n-2)$  is a polynomial of degree **4**.

**e** Define  $G: [1..12] \rightarrow \mathbb{N}$  where  $G(n)$  is the number of letters in the  $n^{\text{th}}$  Gregorian month. So  $G(2) = 8$ , since the 2<sup>nd</sup> month is "February". The only fixed-point of  $G$  is **9**. The set of posints  $k$  where  $G^{\circ k}(12) = G^{\circ k}(7)$  is **[4 .. ∞)**.

[January, February, March, April, May, June, July, August, September, October, November, December]

**Soln: Month-map questions.** Here is the orbit-structure of  $G$ :



OYOP: In *grammatical English sentences*, write your essay on every *third* line (usually), so that I can easily write between the lines. Start each essay on a new sheet-of-paper. Please number the pages "1 of 57", "2 of 57"... (or "1/57", "2/57"...). I suggest you put your name on each sheet.

**B5:** **i** Give our formal definition of what it means for a  $\pi \in \mathbb{S}_N$  to be an *even* [i.e, +1] or an *odd* permutation [i.e, -1]. This is call the *sign* of  $\pi$ , written,  $\text{Sgn}(\pi)$ .

**ii** Prove,  $\forall \beta, \alpha \in \mathbb{S}_N$ , that  $\text{Sgn}(\beta \circ \alpha) = \text{Sgn}(\beta) \cdot \text{Sgn}(\alpha)$ .

End of Class-B

**B4:** \_\_\_\_\_ 95pts

**B5:** \_\_\_\_\_ 55pts

**Total:** \_\_\_\_\_ 150pts