List of article abstracts

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1. *A counterexample to a positive entropy skew product generalization of the Pinsker Conjecture*, Ergodic Theory and Dynamical Systems **5** (1985), 379–407.

Constructs a K-automorphism which cannot be a factor of any ***simple independent skew product***. Such a skew product has the following form. Start with some Bernoulli transformation $B\colon X \to X$ and a two-set independent partition $P = \langle P_0, P_1 \rangle$. Given an arbitrary ergodic map $T\colon Y \to Y$, possibly of positive entropy, form the skew product $S \coloneqq B \otimes_P T$ which maps

$$x, y \;\mapsto\; Bx, T^i(y) \qquad \text{where } x \in P_i$$

for each point in the product space $X \times Y$.

A technical condition is that P be refined by some independent *generating* partition.

[2] *The commutant is the weak-closure of the powers, for rank-1 transformations*, Ergodic Theory and Dynamical Systems **6** (1986), 363–384.

In the class of rank-1 maps there is a dichotomy. For such a T the centralizer is either *trivial*, consisting of just the powers of T, or is *uncountable*. [The centralizer $C(T)$, also called the commutant, is the semigroup of transformations which commute with T.] This follows from the Weak-Closure Theorem which states that any $S \in C(T)$ is a weak limit of powers of T. Such an S is called a ***generalized power*** of T.

As a consequence, weak-isomorphism between two maps is equivalent to isomorphism if one of these maps is rank-1. In addition, no rank-1 can be a cartesian square and the non-rigid rank-1 maps cannot be cartesian products. Furthermore, if a countable cartesian product

$$S_1 \times S_2 \times \times S_3 \times \ldots$$

is rank-1 then the transformations $\{S_n\}_{n=1}^{\infty}$ must be mutually disjoint in the sense of Furstenberg.

To show that this last result is non-vacuous, the article constructs a weak-mixing countable cartesian product which is rank-1.

§2 shows that any proper factor of a rank-1 must be rigid. [A rigid map is one possessing a never-zero sequence of powers which converge to the identity transformation.] Furthermore, *rank-1 has unique factors:* If $\mathcal{F}$ and $\mathcal{G}$ are two T-invariant sub-σ-algebras such that the factor $T|_{\mathcal{F}}$ is isomorphic to $T|_{\mathcal{G}}$ then $\mathcal{F} = \mathcal{G}$.

[3] *For mixing transformations rank*$(T^k) = k \cdot$ *rank*(T), Israel J. Math. **56** (1986), 102–122.

Using a coding/blocking argument this article shows that for any map T with zero rigidity, in particular for any mixing T, the rank of T^k equals $k \cdot \operatorname{rank}(T)$ for any natural number k. Here, ***rank*** means what is sometimes called "uniform rank".

Map T has ***zero rigidity*** if for any sequence of integers $n(i) \to \infty$ and for any $\varepsilon > 0$ there exists some set A such that

$$\liminf_{i\to\infty} \mu\big(A \cap T^{n(i)} A\big) < \varepsilon \cdot \mu(A)\,.$$

[6] *A lower bound on the rank of mixing extensions*, Israel J. Math. **59** #3 (1987), 377–380.

If T has zero rigidity and S is a k-point extension of T then $\operatorname{rank}(S) \geq k \cdot \operatorname{rank}(T)$. This is a second application of the coding argument of the above paper.

[4] [with N. Friedman and P. Gabriel], *An invariant for rank-1 rigid transformations*, Ergodic Theory and Dynamical Systems **8** (1988), 53-72.

Associated to a rigid rank-1 transformation T is a semigroup $\mathcal{L}(T)$ of natural numbers, closed under factors. If $\mathcal{L}(S) \neq \mathcal{L}(T)$ then S and T cannot be copied isomorphically onto the same space so that they commute. If $\mathcal{L}(S) \not\supset \mathcal{L}(T)$ then S cannot be a factor of T. For each semigroup L we construct a weak mixing S such that $\mathcal{L}(S) = L$. The S where $\mathcal{L}(S) = \{1\}$, despite have uncountable commutant, has no roots. The technique appears to be novel: To establish that S has no cube root, for instance, one need but arrange that the commutant of S^3 be non-Abelian.

Preceding and preparing for this example are two others: An uncountable Abelian group G of weak mixing transformations for which any two (non-identity) members have identical self-joinings of all orders and powers. The second example, to contrast with the rank-1 property that the weak essential commutant must be the trivial group, is of a rank-2 transformation with uncountable weak essential commutant.

[5] *Joining-rank and the structure of finite rank mixing transformations*, J. d'Analyse Math. **51** (1988), 182–227.

This article presents a new isomorphism invariant of zero-entropy maps, called ***joining-rank***. Written $\operatorname{jr}(T)$, it is a value in $\mathbb{N} \cup \{\infty\}$. The depth of factors of T, and the size of its essential commutant, $\operatorname{EC}(T)$, are upper bounded by $\operatorname{jr}(T)$. If T is mixing then $\operatorname{jr}(T) \leqslant \operatorname{rank}(T)$. For T with finite joining-rank we obtain a structure theorem for the commutant group of T; it is a certain twisted product (a "carry product") of $\mathbb{Z}$ with $\operatorname{EC}(T)$. As for T itself, it must be an m-point extension of the nth power of a prime transformation S having trivial commutant. Also, $\operatorname{jr}(T) = m \cdot n \cdot \operatorname{jr}(S)$.

The ***covering number***, $\kappa(T)$, is a number in $[0,1]$ obeying $1/\kappa(T) \leq \operatorname{rank}(T)$. Let $\alpha(T) \in [0,1]$ denote the degree of partial mixing of T. Then $\operatorname{jr}(T)$ is dominated by

$$1 \big/ \left[\kappa(T) + \alpha(T) - 1\right].$$

In particular, a rank-1 T with partial mixing exceeding $\frac{1}{2}$ has minimal self-joinings.

Combined with Kalikow's deep theorem that, when T is rank-1, 2-fold mixing implies mixing of all orders, our technique yields that a mixing such T has minimal self-joinings of all orders. Thus T may be used as the seed for Rudolph's counterexample machine.

[§1 is written as an extensive introduction suitable for ergodic theory graduate students. It proves most of the elementary facts about rank and covering-rank, partial mixing/rigidity and joinings. §2 contains technical joining proofs of the functorial properties of joining-rank.

A practical criterion that a transformation has finite joining-rank is provided by §3 and well-known maps are shown to fulfill it. Once a map has finite joining-rank, §4 shows that essentially algebraic techniques derive further properties; in particular, if a mixing T has rank less or equal to 5 then its commutant is perforce Abelian. The main result of §4 is a structure theorem which says that if $\operatorname{jr}(T) < \infty$ then T is a finite extension of a power of a prime transformation with trivial commutant.

§5 constructs a map (an algebraic extension of Ornstein's or of Chacón's map) with joining-rank three but with trivial commutant. This is a counterexample to the conjecture the the finite extension in the *Rudimentary Structure*

Theorem must be a sequence of group extensions. The article concludes with §6 with an algebraic structure theorem for the commutant group of T, when $\mathrm{jr}(T) < \infty$.]

[7] [with B. Weiss], *A group rotation factor of a non-rigid rank-1 map*, Dynamical Systems (Proceedings, U. of Maryland), ed. J.C. Alexander, Springer Lecture Notes in Mathematics #1342, 1986–87, pp. 425–439.

Using cutting&stacking along with a joining argument, we construct a transformation on the non-rigid side of the dichotomy in Rank-1 which has an irrational rotation as factor. This gives an example where the canonical group homomorphism of the commutant of a rank-1 map to the commutant of a factor is not surjective.

[8] *Lightly mixing is closed under countable products*, Israel J. Math. **62** #3 (1988), 341–346.

Transformation T is ***lightly mixing*** if for each pair of non-null sets A, B the quantity

$$\liminf_{n\to\infty} \mu\big(A \cap T^{-n} B\big)$$

is positive. This article answers a question of Nat Friedman by showing that the countable cartesian product of lightly mixing transformations is itself lightly mixing.

As a byproduct, we show that the class of lightly mixing maps is closed under projective limit. Additionally, it is observed that the "Rényi" version of lightly mixing implies lightly mixing.

[9] *An obstruction to K-fold splitting*, (in honor of Dorothy Maharam Stone), ed. R.D. Mauldin, R.M. Shortt, C.E. Silva, vol. 94, American Mathematical Society, 1989, pp. 171–175.

Let $\mathrm{mix}(T)$ denote the partial mixing of T and let $\mathrm{rig}(T)$ denote its degree of partial rigidity. We show that if

$$\big[\mathrm{rig}(T)\big]^K + \big[\mathrm{mix}(T)\big]^K > 1 \qquad (*)$$

then T cannot be a K-fold cartesian product; indeed, it can have no factor which is. This remains true if (*) holds where the partial rigidity and mixing are computed along any subsequence. As a consequence, if

$$\mathrm{rig}(T) + \sqrt{\mathrm{mix}(T)} > 1\,,$$

then no power of T can have a cartesian product factor.

[10] *A map with topological minimal self-joinings in the sense of del Junco*, Ergodic Theory and Dynamical Systems **10** (1990), 745–761.

Constructs a homeomorphism T of a zero-dimensional space satisfying del Junco's definition of 2-fold topological minimal self-joinings. To wit: Whenever x and y are not in the same T-orbit, the $T{\times}T$-orbit of (x, y) is dense; also, each non-zero power of T is minimal.

Conversely, by means of a generalization of a theorem of Schwartzman, the article answers negatively a question of del Junco by showing that 4-fold topological minimal self-joinings cannot exist.

[11] *Closure properties of the class of uniform sweeping-out transformations*, Illinois J. Math. **36** #2 (1992), 233–237.

Friedman defines T to be ***uniformly sweeping out*** if for every ε and set A of positive mass, there exists N such that

$$\mu\bigg(\bigcup_{k\in\mathbb{K}} T^k A\bigg) > 1 - \varepsilon$$

for each collection $\mathbb{K}$ of integers with $|\mathbb{K}| \geq N$. This property is implied by mixing. The converse is not known.

Extending the technique of 8 above, this article shows that the class of uniformly sweeping out maps is closed under countable direct product and under inverse limit.

[12] [with Jean-Paul Thouvenot], *A canonical structure theorem for finite joining-rank maps*, J. d'Analyse Math. **56** (1991), 211–230.

Making use of Furstenberg's notion of a "relative weak-mixing extension", this article makes the structure theorem of 5 canonical. For a transformation T of finite joining-rank ($\mathrm{jr}(T) < \infty$) there is a unique triple $\langle\ e, p, S\rangle$, where e and p are natural numbers and S is a map with *minimal self-joinings*, such that the given T is an e-point extension of S^p.

[13] *Ergodic properties where order 4 implies infinite order*, Israel J. Math. **80** (1992), 65–86.

The motivation is Steven Kalikow's celebrated result that for rank-1 $\mathbb{Z}$-actions, 2-fold mixing implies 3-fold mixing. Here, we are obtain weaker, but more general results that for certain properties which can be characterized in terms of joinings, if the action has the property for order 4 then it has it for all orders.

Suppose $\mathbb{G}$ is an amenable group. A general joining argument shows that a $\mathbb{G}$-action which has 4-fold minimal self-joinings has minimal self-joinings of all orders. The same argument shows that 4-fold simplicity implies simplicity of all orders, and that 2-fold cartesian disjointness (in the sense of Furstenberg) implies cartesian disjointness of all orders.

A corollary is that if a amenable group action, which is close to being "rank-1" in a certain sense, is 4-fold mixing then it is mixing of all orders. This result is new even for $\mathbb{Z}$-actions.

[14] [with N. Friedman], *A Bernoulli which is a generalized power of a rank-1 map*, In preparation..

We construct a rank-1 transformation T (which of necessity has zero-entropy) possessing a transformation of infinite entropy as a generalized power. The construction is then modified to produce a Bernoulli transformation which is a generalized power of T.

[15] [with N. Friedman], *Rank one lightly mixing*, Israel J. Math. **73** #3 (1991), 281–288.

A lightly-mixing rank-one map T which is not partially-mixing (and is not 3-fold lightly-mixing) is constructed by means of cutting&stacking, thus answering a question of [8]. This shows that the dynamical properties *weak-mixing, mildly-mixing, lightly-mixing, partially-mixing, mixing* –which get strictly stronger from left to right– are distinct even in the restricted class of rank-1 maps.

[16] *Dilemma of the Sleeping Stockbroker*, Amer. Math. Monthly **99** #4 (1992), 335–338.

Constructs a real-valued stationary stochastic process $\{X_n\}_{n=-\infty}^{\infty}$ which is deterministic in a strong sense: Knowing the value of the pair X_0, X_1 determines the entire process.

Conversely, the process is random in a strong sense. If $\{n_i\}_{i=-\infty}^{\infty}$ has no two consecutive terms, $n_{i+1} > n_i + 1$, then $\{X_{n_i}\}_{i=-\infty}^{\infty}$ is an independent Bernoulli process.

[17] *Three Problems in search of a Measure*, Amer. Math. Monthly **101** #7 (1994), 609–628.

(In 1995, this won a award for mathematical exposition.) Uses invariant measures to prove Poncelet's Theorem, and solve the elementary Tarski's Plank Problem, and Gelfand's Question. Although this is primarily an expository article, it contains a new proof of Poncelet's Theorem.

[18] *Billiards inside a Cusp*, Math. Intelligencer **17** #1 (1995), 8–16.

Uses invariant measures to answer a question of David Feldman concerning billiards on a table bounded by the curves $y = \pm\frac{1}{x}$. It proves that a measure-preserving flow on a pinched-cusp, even of infinite measure, satisfies Poincaré recurrence. Although this result appears to be new, this article is primarily expository, and is a companion to the preceding article.

[19] [with S. Kalikow], *A countably-valued sleeping stockbroker process*, J. of Theoretical Probability **7** #4 (1994), 703–708.

This answers a question of *Dilemma of the Sleeping Stockbroker*.

The article exhibits a stationary countably-valued process $\{V_n\}_{-\infty}^{\infty}$ which is deterministic, but which is non-deterministic in the sense that whenever $\ldots n_{-2} < n_{-1} < n_0 < n_1 < \ldots$ are indices with no two consecutive, then $\{V_{n_i} \mid i \in \mathbb{Z}\}$ is an independent process.

In addition, although $n \mapsto V_n$ is deterministic, its time reversal $n \mapsto V_{-n}$ is not deterministic.

[20] [with Mustafa Akcoglu], *An example of pointwise non-convergence of iterated Conditional Expectation Operators*, Israel J. Math. **94** (1996), 179–188.

Given ε, this constructs a sequence $\mathcal{F}_1, \mathcal{F}_2, \ldots$ of Borel sub-sigma-algebras on the unit interval with the following property. Suppose the identity function $f(x) = x$ is transformed by successive conditioning on $\mathcal{F}_1$, then $\mathcal{F}_2$, … , then $\mathcal{F}_n$, … . Then the limsup, with respect to n, will exceed (pointwise almost-everywhere) $1 - \varepsilon$ and its liminf will be less than ε.

The sequence of functions also will fail to converge in the $\mathbb{L}_2$-norm. This contrasts with the long-open conjecture that if all the $\mathcal{F}_n$ come from a finite set of sigma-algebras, then the resulting sequence of functions must converge in $\mathbb{L}_2$.

[21] *An answer to M. Gordin's homoclinic question*, International Mathematics Research Notices no. 5 (1997), 203-212.

This constructs a zero-entropy transformation T whose homoclinic group, $\mathcal{H}[T]$, acts ergodically, thus answering a question of Misha Gordin. We extend the method to build a rigid map with the analogous property off of a density-zero sequence of times. Gordin defines a transformation S is ***homoclinic*** with respect to T if the following limit and equality exist.

$$\lim T^p S T^{-p} = Id, \qquad \textit{as } p \to \infty, \textit{ and as } p \to -\infty.$$

Gordin, in an earlier paper, showed that if T's homoclinic group acts ergodically, then a certain spectral condition on any $\mathbb{L}_2$ function f is sufficient to guarantee that f satisfies the Central Limit Theorem.

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