Staple!

Team A____

Number Theory MAS4203 8430 — **Home-A** — Prof. JLF King, *Touch:* 2Jul2018

Hello. Essays violate the CHECKLIST at *Grade Peril*! Exam is due by **3PM, Thursday, 08Feb2007**, slid **completely under** my office door, Little Hall 402.

Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

A1: Show no work.

___ ___ a) So $z=$ __________ is the smallest natnum satisfying

$$z \equiv_7 -2, \quad z \equiv_8 -1, \quad z \equiv_{11} 5, \quad z \equiv_{15} 12.$$

___ ___ b) And $y=$ __________ is the smallest natnum with

$$y \equiv_{20} 1, \quad y \equiv_{15} 11, \quad y \equiv_{12} 5.$$

___ ___ c) Let $G := \text{Gcd}(70, 42, 30)$; so $G=$ __________.
Use the LBolt Alg twice to find three integers with ______ $\cdot 70 +$ ______ $\cdot 42 +$ ______ $\cdot 30 = G$.

___ ___ d⁺) [Changed!] As polynomials in $\mathbb{Z}_7[x]$, let

$$B(x) := x^4 - 2x^3 + x - 2;$$
$$C(x) := x^3 + 3x^2 - 3x.$$

Write t.fol polys, using coeffs in [-3 .. 3]. Compute quotient and remainder polynomials,
$q(x)=$ __________ & $r(x)=$ __________,
with $B = [q \cdot C] + r$ and $\text{Deg}(r) < \text{Deg}(C)$.

___ ___ e⁺) With B,C from above, polys in $\mathbb{Z}_7[x]$: Let D be $\text{Gcd}(B, C)$. Write these three polys using coeffs in [-3 .. 3]: The **monic** $D(x) =$ __________.
Compute polys $S(x)=$ __________,
$T(x)=$ __________ st. $[S \cdot B] + [T \cdot C] = D$.

___ ___ f) $\varphi(121000) =$ __________.
Express your answer a product $p_1^{e_1} \cdot p_2^{e_2} \cdot \ldots$ of primes to posint powers, with $p_1 < p_2 < \ldots$.

___ ___ g) Easily, $\varphi(25)=$ ______. Consequently, $27^{2006} \equiv_{25}$ ______ $\in [0..25)$. [*Hint:* Fermat, Euler, working mod 25.]

Essay questions: Type in complete sentences and also fill-in the blanks. Each essay starts a new page.

A2: Show the orbit $n \mapsto \langle 7^{2^n} \rangle_{77}$, for $n \in [0..10]$. In binary, 707 is __________. So $k =$ ______ $\in [0..77)$, where $k \equiv_{77} 7^{707}$; briefly show how computed.

A3: i) Prove that Min $^{\text{d}}/_{\text{o}}$ ("distributes over") Max. (Do not bother to prove that Max $^{\text{d}}/_{\text{o}}$ Min; you may use that for free in part (ii).)

ii) Acting on $\mathbb{Z}_+$, prove that Gcd $^{\text{d}}/_{\text{o}}$ Lcm. One approach is part (i) combined with FTArithmetic.

A4: Use Wilson's Thm to *prove* #43a P.63 of Strayer. Thus $2 \cdot [99!] \equiv_{103}$ ______ $\in (-50..51]$.

A1:	___ ___ ___	150pts
A2:	___ ___	65pts
A3:	___ ___	65pts
A4:	___ ___	65pts
Total:	___ ___ ___	345pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

__________ Ord: ____

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