Staple!

Team: ____

Moda
MAA4226 MAA5228

Home-A

Prof. JLF King
Touch: 6May2016

Hello. Our take-home is due at the **BoC, Monday, 11 Oct. 2010**. Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

A1: Show no work. Fill-in *all* blanks *on <u>this</u> sheet*! (*Handwriting is fine; don't bother to type*).

___ ___ **a** Let $S := \{q \in \mathbb{Q}_+ \mid 5 \leqslant q^2 < 9\}$. Then:

$\mathrm{Cl}_{\mathbb{R}}(S)=$ ________. $\mathrm{Itr}_{\mathbb{R}}(S)=$ ________.

$\mathrm{Cl}_{\mathbb{Q}}(S)=$ ________. $\mathrm{Itr}_{\mathbb{Q}}(S)=$ ________.

___ ___ **b_{14}** Let $S := \{q \in \mathbb{Q}_+ \mid 5 \leqslant q^2 < 9\}$. Then:

$\partial_{\mathbb{R}}(S)=$ ________. $\partial_{\mathbb{Q}}(S)=$ ________.

___ ___ **c_{24}** Suppose that $U, V_1, V_2, \ldots$ are $\mathbb{R}$-open-sets, and $E, K_1, K_2, \ldots$ are $\mathbb{R}$-closed-sets. Circle those of the following sets which are <u>guaranteed</u> to be $\mathbb{R}$-*closed*.

$K_1 \smallsetminus E$. $\quad \partial_{\mathbb{R}}(E) \cap \mathrm{Itr}_{\mathbb{R}}(E)$. $\quad \partial_{\mathbb{R}}(E) \cup \mathrm{Itr}_{\mathbb{R}}(E)$.

$\mathbb{R} \smallsetminus \big[\bigcup_{n=1}^{\infty} V_n\big]$. $\quad \bigcup_{n=1}^{\infty} \mathrm{Cl}_{\mathbb{R}}(V_n)$. $\quad [\mathrm{Itr}_{\mathbb{R}}(E) \cap V_1]^c$.

___ ___ **d_{10}** Give an example of a set,

$$\Big\{ \underline{\qquad\qquad} \in \mathbb{R} \;\Big|\; \underline{\qquad\qquad\qquad\qquad} \Big\},$$

which has $5, 8, 9 \in \mathbb{R}$ as its <u>**only**</u> $\mathbb{R}$-cluster points.

___ ___ **e** Define $\Omega :=$ ________ $\subset \mathbb{R}$ st. the Ω-*closed* ball $C := \Omega\text{-CldBal}_5(0) =$ ________ satisfies

$C \supsetneqq \mathrm{Itr}_{\Omega}(C)=$ ________ $\supsetneqq \Omega\text{-Bal}_5(0)=$ ________.

Essay questions: For each question, carefully type a triple–spaced essay solving the problem.
Each essay starts a new page.

A2: For a subset S of MS (X, m), let S' comprise the X-cluster-pts of S. For the three parts below, *Illustrate* your various sets with useful, colored pictures!

i *Prove*: THEOREM: *Always, S' is X-closed.*

ii Give an example [with proof, natch'] of a set $S \subset \mathbb{R}$ (m := usual metric) so that $S' \not\subset S$ and $S' \not\supset S$.

iii For $N \in \mathbb{N}$, say an $S \subset \mathbb{R}$ is ***N*-good** if

$$S \supsetneqq S' \supsetneqq S'' \supsetneqq \cdots \supsetneqq S^{(N)} = S^{(N+1)}.$$

In $\mathbb{R}$, give examples of a 1-good set A, a 2-good set B, and a 3-good set C. *Then*: For each natnum N construct, with proof, an N-good set $D_N \subset \mathbb{R}$.

A3: A MS (Y, d) is ***sequentially compact*** if each $\vec{\mathbf{b}} \subset Y$ has a subsequence $\vec{\mathbf{a}} \subset \vec{\mathbf{b}}$ which is Y-convergent. Prove that Y is sequentially-cpt (seq-cpt) IFF each infinite subset $E \subset Y$ has a cluster-point in Y.

End of Home-A

A1:	___ ___	88pts
A2:	___ ___	85pts
A3:	___ ___	85pts
Poorly stapled, or missing ordinals :	___	−5pts
Missing names, or honor sigs :	___	−5pts
Total:	___ ___ ___	258pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

________ Ord:

________ Ord:

________ Ord: