

NT-Cryptography
MAT4930 7554

Home-A

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Touch: 4Aug2016

BoC, Monday, 10Feb2014, Please *fill-in* every *blank* on this sheet.

A1: Show no work. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a $N := \varphi(100) = \dots$. So $\varphi(N) = \dots$.
EFT says that $3^{1621} \equiv_N \dots \in [0..N)$. Hence (by EFT) last two digits of $7^{[3^{1621}]}$ are \dots .

b Number $M := 229$ is prime. PoP-factor $\varphi(M)$ as \dots . Compute the multiplicative-order, $\text{Ord}_M(-5) = \dots$. [Hint: Use the Descent Alg.]

c As polynomials in $\Gamma := \mathbb{Z}_7[x]$, let

$$B(x) := x^4 - 2x^3 + x - 2;$$

$$C(x) := x^3 + 3x^2 - 3x.$$

Write t.fol polys, using coeffs in $[-3..3]$; use \equiv for equality in \mathbb{Z}_7 and in Γ . Compute quotient and remainder polys, $q(x) \equiv \dots$ & $r(x) \equiv \dots$, with $B \equiv [q \cdot C] + r$ and $\text{Deg}(r) < \text{Deg}(C)$.

Let $D := \text{Gcd}(B, C)$. **Monic** $D(x) \equiv \dots$.
Compute polys $S(x) \equiv \dots$,
 $T(x) \equiv \dots$ st. $[S \cdot B] + [T \cdot C] \equiv D$.

OYOP: Your 2 essay(s) must be TYPESET, and Double or Triple spaced. Use the Print/Revise cycle to produce good, well thought out, essays. Start each essay on a **NEW** sheet of paper.

Do **not** restate the problem; just solve it.

A2: Consider $M := p \cdot q = 40349$, where $p < q$ are primes. Your mole in King's organization finds out that $F := \varphi(M) = 39936$. Use the method from class, showing all the steps, to compute the factors

$$p = \dots < q = \dots$$

A3: The RSA system uses a modulus $M := p \cdot q$, where $p < q$ are primes. A message in an element of \mathbb{Z}_M . In class, we required this elt be coprime to M , but this is not necessary. So: In our text, solve [prove] problem **3.2**, on P.176. It refers to Proposition 3.4, on P.116.

End of Home-A

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|--|---------------|-------|--------|
| | A1: | _____ | 80pts |
| | A2: | _____ | 80pts |
| <i>Poorly stapled, or missing names or Honor code:</i> | A3: | _____ | 105pts |
| | | _____ | -15pts |
| <i>Not typed/double-spaced:</i> | | _____ | -25pts |
| | Total: | _____ | 265pts |

HONOR CODE: "I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)." *Name/Signature/Ord*

Ord: _____

Ord: _____

Ord: _____