Staple!

Team: A

Abstract Algebra MAS4301 09B1 — **Home-A** — Prof. JLF King 07Feb2008

Hello. Essays violate the CHECKLIST at *Grade Peril*! Exam is due by **BoC, Monday, 23Sep2019** with ATP!. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0 \neq$ *Empty-word.*.

Let F and R be the ***flip*** and ***rotation*** in the dihedral group $\mathbb{D}_N$, with $F^2 = e$, $R^N = e$ and $RFRF = e$. Use R^j and R^jF as the standard form of each element in $\mathbb{D}_N$.

Use $\mathbb{Z}_N$ to denote the cyclic group of order N.

Fill-in *all* blanks (*handwriting; don't bother to type*) on this sheet **including** the blanks for the essay questions!

A1: Show no work.

a) Mod K:=4301, the recipr. $\langle \frac{1}{237} \rangle_K =$ ______ $\in [0..K)$. [*Hint:* ↯] So $x =$ ______ $\in [0..K)$ solves $4 - 237x \equiv_K 1$.

b) $G := (\mathbf{U}(23), \cdot, 1)$ is cyclic. The smallest generator is ______ $\in [2..21]$. And G has ______ many generators.

c) In $\mathbb{S}_4$, the centralizer of $\mathbf{q} := (1\ 2)(3\ 4)$ has ______ many elts. In $\mathsf{C}(\mathbf{q})$, the number of elements of each cycle-signature is: $\lceil 1^4 \rfloor$: ______. $\lceil 1^2, 2^1 \rfloor$: ______. $\lceil 1^1, 3^1 \rfloor$: ______. $\lceil 2^2 \rfloor$: ______. $\lceil 4^1 \rfloor$: ______.

d) In $\mathbb{S}_4$, the subgp, H, generated by $y := (1\ 2)(3\ 4)$ and $z := (2\ 4\ 3)$ has ______ many elements.

e) Elt $\alpha^3 = (6\,4\,1\,0\,3\,5\,2) \in \mathbb{S}_7$. So $\alpha =$ ______

f) Perm $\beta \in \mathbb{S}_{15}$ has sig $\lceil 5^3 \rfloor$. It has ______ many sqroots with sig $\lceil 5^3 \rfloor$, and ______ with sig $\lceil 10^1, 5^1 \rfloor$.

g) Circle the one group which is *not* isomorphic to any of the others:

$\mathbb{Z}_2 \times \mathbb{Z}_6$ $\quad \mathbb{D}_6 \quad$ $\mathbf{U}(13)$ $\quad \mathbb{Z}_4 \times \mathbb{Z}_3 \quad$ $\mathbb{S}_3 \times \mathbb{Z}_2$.

The remaining four groups can be paired into two isomorphic pairs. Underline the cyclic pair.

h) In $\mathbb{S}_{11}$, the maximum possible order of an element is $\text{MaxOrd}(\mathbb{S}_{11}) = \text{LCM}($ ______ $) =$ ______.

For the essay questions, carefully TYPE, *double-spaced, grammatical solns. I suggest* LaTeX, *but other systems are ok too.*

Fill-in all blanks. Each essay starts a new page.

A2: Produce (with proof, natch') a finite group G and explicit elts $\boldsymbol{x}, \boldsymbol{y} \in G$ with *different prime* orders $p \neq q$, so that $\text{Ord}(\boldsymbol{xy}) \perp p \cdot q$. [*Hint:* Necessarily, $\boldsymbol{x} \neq \boldsymbol{y}$.]

A3: Group $\mathbb{D}_5$ has ______ many automorphisms of which ______ are inner-auts. Exhibit an *outer*-aut, defined by $\alpha(R) :=$ ______ and $\alpha(F) :=$ ______. [Use form $R^J F^K$.] Prove that your defn extends to an automorphism. Prove that your α is **not** an inner-automorphism.

A4: *Prove or disprove*: Multiplicative groups $G := \mathbf{U}(20)$ and $H := \mathbf{U}(24)$ are isomorphic.

End of Home-A

A1:	___ ___ ___	150pts
A2:	___ ___	35pts
A3:	___ ___	75pts
A4:	___ ___	35pts
Total:	___ ___ ___	295pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

______ Ord: ____

______ Ord: ____

______ Ord: ____