Calc 2 MAC2312 — **Home-A** — Prof. JLF King 31Aug2015

Hello. Take-home A is due (at the beginning of class) on **Friday, 29 Sept; 1995**.

A1:

a $\left[\sqrt{3}^{\sqrt{2}}\right]^{\sqrt{8}}=$ .... $\log_3(9^4)=$ ....

b For $x>0$, let $B(x):=x^{\log(x)}$. Its derivative is $B'(x)=$ ....

c Let $y=f(x):=\left[[x\cdot\sqrt[3]{x}\,]-2\right]/4$. Its inverse-function is $f^{-1}(y)=$ ....

d Let $g(x):=x^3+x$. Then $g^{-1}(10)=$ .... and $[g^{-1}]'(10)=$ ....

e Let $y=h(x):=\int_5^x \frac{1}{t}\,dt$. Then $h^{-1}(y)=$ ....

f Define fncs $E,G\colon[1..12]\to[1..12]$ where $E(n)$ is the number of letters in English name of n [so $E(9)=4$, since "nine" has 4 letters] and $G(k)$ counts the #letters in the k^{th} Gregorian month. So $G(2)=8$, since the 2^{nd} month is "February"; 8 letters. Now: $E\Big(E\big(E(11)\big)\Big)=$ ....

Statement "$E\circ G = G\circ E$" is circle *T* *F*

A2: Define binary operation "♡", on pair of reals, by $x\heartsuit y := xy+y$. Prove or disprove (i.e give an explicit counterexample): "♡" is associative.

On the set of postive real numbers, define "★", a binary operation, by $a\bigstar b := a^{\log(b)}$. Prove or disprove: "★" is associative.

A3: Please write up a solution to #64 P.427 of our text. Draw a careful picture of the situation.

A4: We explore the exponentiation operation. All the numbers in the questions below are *positive*.

i Put the correct relation, "<", "=", ">" between the following two quantities: π^{e} and e^{π}.

Of course you can make a reasonable guess using a hand-held approximator –but can you simply make a rigorous argument *without* a crutch, just using the properties of log() that you know together with the fact that $\pi > e$?

ii Define these two numbers,

$$a := 2.\overbrace{000\ldots 005}^{1000\text{ zeros}} \quad\text{and}\quad b := 2.\overbrace{000\ldots 006}^{1000\text{ zeros}}.$$

Which number, a^b or b^a is larger, or are they equal? Carefully justify your answer.

iii Here x and y denote positive numbers. For each x, how many positive reals y are so that

1: $$x^y = y^x$$

is satisfied? Of course, $y:=x$ is one such solution.

How many values (and what are they) of x satisfy that the *only* solution to eqn (**??**) is $y=x$?

End of Home-A

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