

Combinatorics
MAD4203 3214

Home-A

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23Sep2017

End of Home-A

Due **BoC, Wednesday, 27Sep2017**, Please *fill-in* every blank on this sheet. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. *In grammatical English sentences*, TYPE the essay on every *third line* (usually), so that I can easily write between the lines. Do **not** restate the question.

A1: _____ 95pts

A2: _____ 65pts

Total: _____ 160pts

A1: Over a 29 day month, Combinatorist Cathy posts at least one soln per day, for a total of 45 solns. PROVE:

There is a period of consecutive days over which she posted exactly 12 solutions.

NOTE: In your proof, let s_n denote the number of solns posted that month by the end of day n . By hyp., then,

$$1 \leq s_1 < s_2 < \dots < s_{29} = 45.$$

Let $t_n := 12 + s_n$. Using this notation, write a complete, rigorous proof, proving any lemmas you need/want.

Call a natnum g **good** if there is an interval of consecutive days over which Cathy posts exactly g solns. What can you tell me about the set, \mathcal{G} , of good g ?

HONOR CODE: "I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)." Name/Signature/Ord

Ord:

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Ord:

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Ord:

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Note: Staple this problem-sheet to your essay. The problem-sheet is the first page of what you hand in.

A2: Henceforth, show no work. Simply fill-in each blank on the problem-sheet.

a The coeff of $x^7 y^{12}$ in $[5x + y^3 + 1]^{30}$ is _____

[You may write in form number times multinomial-coeff. You can leave the multinomial-coeff as such, or write ITOF factorials.]

b Compute the real $\alpha =$ _____ such that

$$3^\alpha \cdot \sum_{k=0}^{4000} \binom{4000}{k} 2^k = \sum_{j=0}^{2017} \binom{2017}{j} 8^j.$$

[Hint: The Binomial Theorem]

c The number of ways of picking 51 objects from 70 types is $\left[\begin{matrix} 51 \\ 70 \end{matrix} \right]_{\text{Binom}}^{\text{coeff}}$. And

$$\left[\begin{matrix} 51 \\ 70 \end{matrix} \right] = \left[\begin{matrix} N \\ T \end{matrix} \right], \text{ where } N = \text{_____} \neq 51, \text{ and } T = \text{_____}.$$

d Given sets with cardinalities $|B| = 8$ and $|E| = 5$, the number of non-constant fncs in B^E is _____