Combinatorics MAD4203 3214 | **Home-A** | Prof. JLF King 23Sep2017

Due **BoC, Wednesday, 27Sep2017**, Please *fill-in* every *blank* on this sheet. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. *In grammatical English* **sentences**, TYPE *the essay on every* **third** *line* (usually), *so that I can easily write between the lines. Do* **not** *restate the question.*

A1: Over a 29 day month, Combinatorist Cathy posts at least one soln per day, for a total of 45 solns. PROVE:

> *There is a period of consecutive days over which she posted exactly 12 solutions.*

NOTE: In your proof, let s_n denote the number of solns posted that month by the end of day n. By hyp., then,

$$1 \leqslant s_1 < s_2 < \ldots < s_{29} = 45\,.$$

Let $t_n := 12 + s_n$. *Using this notation, write a complete, rigorous proof, proving any lemmas you need/want.*

Call a natnum g ***good*** if there is an interval of consecutive days over which Cathy posts exactly g solns. What can you tell me about the *set*, $\mathcal{G}$, of good g?

A2: *Henceforth, show no work. Simply fill-in each blank on the problem-sheet.*

a The coeff of $x^7 y^{12}$ in $[5x + y^3 + 1]^{30}$ is $5^7 \cdot \binom{30}{7,\,4,\,19}$, *since* $\frac{12}{3}$ *is 4* .

b Compute the real $\alpha=$ ________ such that

$$*: \qquad 3^\alpha \cdot \sum_{k=0}^{4000} \binom{4000}{k} 2^k \quad = \quad \sum_{j=0}^{2017} \binom{2017}{j} 8^j\,.$$

[*Hint:* The Binomial Theorem]

Binom Soln: LhS(∗) equals $3^\alpha \cdot [2+1]^{4000} = 3^{\alpha+4000}$. RhS(∗) equals $[8+1]^{2017} \overset{\text{note}}{=\!=} 3^{[2\cdot 2017]}$. Consequently,

$$\alpha \;=\; [2\cdot 2017] - 4000 \;=\; 34\,.$$

c The number of ways of picking 51 objects from 70 types is $\left[\!\left[\begin{matrix}51\\70\end{matrix}\right]\!\right] \overset{\text{Binom}}{\underset{\text{coeff}}{=\!=}} \Big(\qquad\qquad\Big)$. And

$\left[\!\left[\begin{smallmatrix}51\\70\end{smallmatrix}\right]\!\right] = \left[\!\left[\begin{smallmatrix}N\\T\end{smallmatrix}\right]\!\right]$, where $N=$ ________ $\neq 51$, and $T=$ ________ .

Soln: We use $70-1=69$ separators. So

$$\left[\!\left[\begin{matrix}51\\70\end{matrix}\right]\!\right] = \binom{51+69}{51,\ 69} = \binom{120}{51,\ 69}.$$

This also equals

$$\binom{120}{69\,,51} = \binom{69+[52-1]}{69,\ 52-1},$$

so $N = 69$ and $T = 52$.

d Given sets with cardinalities $|B| = 8$ and $|E| = 5$, the number of non-constant fncs in B^E is $8^5 - 8 = 32760$,

…since there are 8 constant-fncs in B^E.

End of Home-A

A1:	___ ___	95pts
A2:	___ ___	65pts
Total:	___ ___ ___	160pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ....

Ord: ....

Ord: ....

Note: Staple this problem-sheet to your essay. The problem-sheet is the first page of what you hand in.