Linear Alg
MAS3114 2481

Class-A

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For questions A1 and A2 please show no work. Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than $.9797\cdots$.

A1: Let $A := \begin{bmatrix} 2 & 3 & 4 \\ -1 & -1 & 1 \\ -2 & 0 & 1 \end{bmatrix}$, $B := \begin{bmatrix} \sqrt{2} & 0 & \sqrt{2} \\ 0 & -1 & 5 \end{bmatrix}$, and let $\mathbf{u} := \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{w} := \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

a So $AB =$ ________________ . $BA =$ ________________ .

b Circle those of the following vectors which are in $\text{Spn}\{\mathbf{u}, \mathbf{w}\}$.

$$\mathbf{v}_1 := (2, 4, 6) \qquad \mathbf{v}_2 := (2, 4, -6) \qquad \mathbf{v}_3 := (0, 0, 0)\,.$$

For the next two parts, let **AT** mean "Always True", let **AF** mean "Always False", let **Nei** mean "NEIther always true nor always false". Please circle the correct response. Below, $\mathbf{u}, \mathbf{v}, \mathbf{w}$ represent *non-zero* vectors in $\mathbb{R}^4$.

c If $\mathbf{w} \in \text{Spn}\{\mathbf{u}, \mathbf{v}\}$ then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. **AT AF Nei**

If $\mathbf{w} \notin \text{Spn}\{\mathbf{u}, \mathbf{v}\}$ then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent. **AT AF Nei**

d $\text{Spn}\{\mathbf{u},\, \mathbf{v},\, \mathbf{w},\, \mathbf{u} + 2\mathbf{v} + 3\mathbf{w}\}$ is all of $\mathbb{R}^4$. **AT AF Nei**

If none of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ is a multiple of the other vectors, then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is lin. independent. **AT AF Nei**

a The inverse of $\begin{bmatrix} 1 & -6 & -7 \\ & -3 & \\ & & 1 \end{bmatrix}$ is $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$.

f Give an example of two 2×2 matrices $A \neq B$ for which $A^2 = B^2$. $A =$ ____________ and $B =$ ____________ .

g Suppose T is a linear map from $\mathbb{R}^3$ to $\mathbb{R}^2$. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for $\mathbb{R}^3$, and let $\{\mathbf{v}_1, \mathbf{v}_2\}$ be the standard basis for $\mathbb{R}^2$. Suppose that $T(\mathbf{e}_1) = \mathbf{v}_1 - 2\mathbf{v}_2$ and $T(\mathbf{e}_2) = 4\mathbf{v}_2$ and $T(\mathbf{e}_3) = 5\mathbf{v}_1 - 3\mathbf{v}_2$.

Then the matrix of T is: ____________________________ .

h,i Consider these two matrices:

$$R := \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \qquad \text{and} \qquad A := \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}.$$

Determine the matrix $[RA]^{36} =$ ______________________ .

[*Hint:* You don't need multiply matrices. Think of the linear transformations that these matrices represent.]

A2: Let $\mathbf{v}$ be the unit vector in direction $\mathbf{e}_3 + \mathbf{e}_2$ (on the sphere, halfway between $\widehat{\mathbf{k}}$ and $\widehat{\jmath}$). Let R be the transformation which rotates CCW about $\mathbf{v}$ by 90 degrees. Then $R(\mathbf{e}_1) = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ where

$x =$ __________ , $y =$ __________ , $z =$ __________ .

A3: A system of 3 equations in unknowns $x_1, \ldots, x_5$ reduces to the augmented matrix $\begin{bmatrix} 1 & 6 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 8 & 0 \\ 0 & 0 & 0 & 1 & 5 & 0 \end{bmatrix}$.

On your own paper, describe the *general solution* in this form

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \alpha \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \beta \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \gamma \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \ldots$$

where each $\alpha, \beta, \gamma, \delta, \ldots$ is a free variable (either x_1 or… or x_5), and each column vector has specific numbers in it. The *number* of free variables is ______________________ .

A1:	___ ___ ___	180pts
A2:	___ ___	45pts
A3:	___ ___	55pts
Total:	___ ___ ___	280pts

Please PRINT your name and ordinal. Ta:

Ord: ____

Honor Code: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ____