Sets and Logic
MHF3202

Class-A

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Wedn., 03Feb2021

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*

Ord: ....

A1: Short answer. Show no work.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

5 5 **a** [With $\mathcal{P}()$ the *powerset* operator, let $S :=$ 3-stooges.] Then $|\mathcal{P}(S)| =$ ______ and $|\mathcal{P}(\mathcal{P}(S))| =$ ______.

10 10 10 **b** $\forall x,z \in \mathbb{Z}$ *with* $x < z$, $\exists y \in \mathbb{Z}$ *st.:* $x < y < z$. T F
$\forall x,z \in \mathbb{Q}$ *with* $x \neq z$, $\exists y \in \mathbb{R}$ *st.:* $x < y < z$. T F
For all sets Ω, *there exists a fnc* $f:\mathbb{R}\to\Omega$. T F

25 **c** The coeff of x^7y^{12} in $[5x + y^3 + 1]^{30}$ is ______.
[Write your answer as a product of powers and a multinomial. Optionally, you can expand the multinomial as a product of binomials.]

By the way, $\binom{30}{7,\,4,\,19} = \binom{30}{7}\cdot\binom{23}{4}\cdot 1$.

25 **d** Compute the real $\alpha =$ ______ such that

$$*: \quad 3^\alpha \cdot \sum_{k=0}^{4000} \binom{4000}{k} 2^k \;=\; \sum_{j=0}^{1995} \binom{1995}{j} 8^j .$$

[*Hint:* The Binomial Theorem]

30 **e** The physics lab has atomic *zinc*, *tin*, *silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] ______ many possibilities.

This number *also* equals the number-of-ways of picking K candies from L many types of candy, where $K =$ ______ $\notin \{1,6\}$ and $L =$ ______ $\notin \{1,4\}$.

10 10 **f** Complex number $[x + \mathbf{i}y]^2 = -18\mathbf{i}$, for *real numbers* $x>y$, where $x =$ ______ and $y =$ ______.

End of Class-A

A1: ___ ___ ___ 140pts

Total: ___ ___ ___ 140pts