Sets and Logic
MHF3202 2787

Class-A

Prof. JLF King
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A4:

b In $\left[5x^2+4y+z^3+7\right]^{20}$, compute these coeffs:

$\text{Coeff}(x^6z^8) =$ 0, since $3 \nmid 8$. [NB: 0 and *DNE* are different.]

$\text{Coeff}(y^5z^6) =$ $5^0 \cdot 4^5 \cdot 1^2 \cdot 7^{20-[0+5+2]} \cdot \binom{20}{0,5,2,13}$.

[You may write answers as a product numbers, powers and multinomial-coeffs.]

c The physics lab has atomic *zinc*, *tin*, *silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] ______ many possibilities.

Types: Computing, $\left[\!\left[\begin{matrix}4\\6\end{matrix}\right]\!\right] = \binom{4-1+6}{4-1,\ 6} = \binom{9}{3,\ 6} = 84$.

d We consider binrels on $\Omega :=$ Stooges $:= \{M, L, C\}$.

There are $2^{3^2-3} = 2^6 = 64$ **Anti-reflexive** binrels, and

Same $= 2^6$ **Reflexive** binrels,

and $2^{1+2+3} = 2^6$ **Symmetric** binrels. The

number of **strict total-orders** is $3! = 6$.

e The ***Threeish-numbers*** comprise $\mathcal{T} := 1+3\mathbb{N}$.
$\mathcal{T}$-number $385 \overset{\text{note}}{=\!=} 35\cdot 11$ is $\mathcal{T}$*-irreducible*: T F

Irr Soln: *False*; $35 = 7\cdot 5$. So $385 = 7\cdot[5\cdot 11]$ is a non-trivial *Threeish*-factorization of 385.

Threeish $N := 85$ is **not** $\mathcal{T}$*-prime* because $\mathcal{T}$-numbers $J :=$ ______ and $K :=$ ______ satisfy that $N \bullet\!| [J\cdot K]$, **yet** $N \nmid J$ and $N \nmid K$.

Prime Solution: An integer k is ***3Neg*** if $k \equiv_3 -1$ and ***3Pos*** if $k \equiv_3 +1$. Note $85 = 5\cdot 17$ is a product of two 3Neg primes. We simply need to place one prime in J and the other in K. Hence a solution is $(J, K) := (5\cdot 5,\ 17\cdot 17)$.

A more general soln is $(J,K) := (5p,\ 17q)$. where p,q are 3Neg numbers st. $p \nmid 17$ and $q \nmid 5$. Letting $p = q := 2$ yields $(J,K) := (10,\ 34)$ as a smallest soln.

f Sequence $\vec{\mathbf{L}} := (L_n)_{n=0}^{\infty}$ is defined by $L_0 := 5$, $L_1 := 4$, and $\boxed{\forall n\in\mathbb{N}:\ L_{n+2} = L_{n+1} + 6L_n}$. This implies $\boxed{\forall k\in\mathbb{N}:\ L_k = [P\cdot\alpha^k + Q\cdot\beta^k]}$, for real numbers $\alpha=$ ______ $< \beta=$ ______.

Lin-Recurr-Soln: The polynomial of the the recurrence is $x^2 - x + -6$, which factors as $[x - -2]\cdot[x-3]$. Hence $\alpha = -2$ and $\beta = 3$.

OYOP: *In grammatical English **sentences**, write your essay on every **third** line* (usually), *so that I can easily write between the lines.* Please number the pages "1 of 57", "2 of 57"... (or "1/57", "2/57"...) I suggest you put your name on each sheet.

A5: Interval-of-integers $\mathbf{J} := [201\,..\,300)$ has 99 elements. A subset $S \subset \mathbf{J}$ is ***Big*** if $|S| = 51$. Subset $S \subset \mathbf{J}$ is ***Perfect*** if there exist *distinct* members $x,y \in S$ st. $x+y = 500$.

Prove that Big $\Rightarrow$ Perfect. [*Hint:* PHP. *Carefully* specify what your pigeon-holes are.]

A4:	___ ___ ___	120pts
A5:	___ ___	45pts
Total:	___ ___ ___	165pts