

A4:

a Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. **Circle** one:

True! Yes! wH'at S a?sEnTENcE

b In $[5x^2 + 4y + z^3 + 7]^{20}$,

compute these coeffs:

Coeff($x^6 z^8$) = 0, since $3 \nmid 8$. [NB: 0 and DNE are different.]

Coeff($y^5 z^6$) = $5^0 \cdot 4^5 \cdot 1^2 \cdot 7^{20-[0+5+2]} \cdot \binom{20}{0, 5, 2, 13}$.

[You may write answers as a product numbers, powers and multinomial-coeffs.]

c The physics lab has atomic *zinc, tin, silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] many possibilities.

Types: Computing, $\begin{bmatrix} 4 \\ 6 \end{bmatrix} = \binom{4-1+6}{4-1, 6} = \binom{9}{3, 6} = 84$.

d We consider binrels on $\Omega := \text{Stoges} := \{M, L, C\}$.

There are $2^{3^2-3} = 2^6 = 64$ **Anti-reflexive** binrels, and

Same = 2^6 **Reflexive** binrels,

and $2^{1+2+3} = 2^6$ **Symmetric** binrels. The

number of **strict total-orders** is $3! = 6$.

e The *Threeish-numbers* comprise $\mathcal{T} := 1 + 3\mathbb{N}$.

\mathcal{T} -number 385 ^{note} $\equiv 35 \cdot 11$ is \mathcal{T} -irreducible: \mathcal{T} **F**

Irr Soln: *False*; $35 = 7 \cdot 5$. So $385 = 7 \cdot [5 \cdot 11]$ is a non-trivial *Threeish*-factorization of 385.

Threeish $N := 85$ is **not** \mathcal{T} -prime because \mathcal{T} -numbers $J :=$ and $K :=$ satisfy

that $N \blacklozenge [J \cdot K]$, **yet** $N \nmid J$ and $N \nmid K$.

Prime Solution: An integer k is *3Neg* if $k \equiv_3 -1$ and *3Pos* if $k \equiv_3 +1$. Note $85 = 5 \cdot 17$ is a product of two 3Neg primes. We simply need to place one prime in J and the other in K . Hence a solution is $(J, K) := (5 \cdot 5, 17 \cdot 17)$.

A more general soln is $(J, K) := (5p, 17q)$, where p, q are 3Neg numbers st. $p \nmid 17$ and $q \nmid 5$. Letting $p = q := 2$ yields $(J, K) := (10, 34)$ as a smallest soln.

f Sequence $\vec{L} := (L_n)_{n=0}^\infty$ is defined by $L_0 := 5$, $L_1 := 4$, and $\boxed{\forall n \in \mathbb{N}: L_{n+2} = L_{n+1} + 6L_n}$. This implies $\boxed{\forall k \in \mathbb{N}: L_k = [P \cdot \alpha^k + Q \cdot \beta^k]}$, for real numbers $\alpha = \dots < \beta = \dots$.

Lin-Recurr-Soln: The polynomial of the the recurrence is $x^2 - x + -6$, which factors as $[x - -2] \cdot [x - 3]$. Hence $\alpha = -2$ and $\beta = 3$.

OYOP: In *grammatical English sentences*, write your essay on every **2nd or 3rd** line (usually), so that I can easily write between the lines. Please number the pages “1 of 57”, “2 of 57”... (or “1/57”, “2/57”...) I suggest you put your name on each sheet.

A5: Interval-of-integers $\mathbf{J} := [201 .. 300)$ has 99 elements. A subset $S \subset \mathbf{J}$ is **Big** if $|S| = 51$. Subset $S \subset \mathbf{J}$ is **Perfect** if there exist *distinct* members $x, y \in S$ st. $x + y = 500$.

Prove that **Big** \Rightarrow **Perfect**. [*Hint:* PHP. Carefully specify what your pigeon-holes are.]

A4: ___ ___ ___ 120pts

A5: ___ ___ 45pts

Total: ___ ___ ___ 165pts