

Plex
MAA4402 2838

Class-A

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Notation. All sets are subsets of \mathbb{C} .
For sets B and E , the difference set is

$$B \setminus E := \{x \in B \mid x \notin E\}.$$

The complement of E is $E^c := \mathbb{C} \setminus E$.

A1: Short answer. Show no work.

Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a Prof. King wears bifocals, and cannot read small handwriting. Circle one: **True!** **Yes!** **Who??**

b A subset $S \subset \mathbb{C}$ is *path-connected* if

c Complex number $[x + iy]^2 = -9i$, for *real numbers*
 $x =$ and $y =$

d Note $[1 + i]^{166} = [\dots] + i \cdot [\dots]$.
 [Hint: Multiplying complexes multiplies their moduli, and adds their angles.]

e All these sets are non-empty: Sets U and V are open. Sets K , E and E_n are closed. Sets S and T are each connected, i.e path-connected.

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|--|-----------|
| Set $U \setminus K$ is open: | AT AF Nei |
| Set $U \cup K$ is open: | AT AF Nei |
| Set $E \cap K$ is closed: | AT AF Nei |
| Union $\bigcup_{n=1}^{\infty} E_n$ is closed: | AT AF Nei |
| $\exists q \in [S \cap T]$; so $S \cup T$ is connected: | AT AF Nei |

- | | |
|--|-----|
| f The empty-set is connected: | T F |
| Punctured ball $\text{PBal}_2(3i)$ is connected: | T F |
| $\text{Sph}_2(5i) \cap \text{Sph}_2(i)$ is connected: | T F |
| $\text{Sph}_2(4i) \cup \text{Sph}_2(-i)$ is connected: | T F |
| $\text{Sph}_2(5i) \cup \text{CldBal}_2(i)$ is closed: | T F |

g Let $S := \text{PBal}_2(3i)$.

Its boundary

is $\partial(S) =$

[You may use our ball/sphere notation as well as \cup , \cap , complement and set-braces, to describe your answer.]

A2: We've written $h(x + iy) = u(x, y) + iv(x, y)$, giving names to the real and imaginary parts of h .

i Suppose h is differentiable at the point $3 + 2i$. Carefully *state* the Cauchy-Riemann eqns for h at $3 + 2i$.

ii Suppose h is differentiable at a point $z \in \mathbb{C}$. Carefully *derive* the Cauchy-Riemann eqns, directly from the defn of "differentiable".

A1: _____ 185pts

A2: _____ 50pts

Total: _____ 235pts