Moda
MAA4226 MAA5228

Class-A

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Hello. Please write **DNE** in a blank *if* the described object does not exist or if the indicated operation cannot be performed.

A1: Show no work. Fill-in *all* blanks *on this sheet*!

__ __ a) Define $X :=$ .... $\subset \mathbb{R}$ st. the X-open ball $B := X\text{-Bal}_3(0) =$ .... satisfies

$B \subsetneqq \mathrm{Cl}_X(B) =$ .... $\subsetneqq X\text{-CldBal}_3(0) =$ ....

__ __ b) A subset $S \subset \Omega^{\mathrm{MS}}$ is a ***neighborhood*** of point $p \in \Omega$ IFF ....

....

....

In $\Omega := \mathbb{R}$, the set $\mathbb{Q}_+ \cap [5, \infty)$ is a neighborhood…

… of 6: **True** **False**. … of 5: **True** **False**.

__ __ c) Let $\mathbf{v} := (2, \text{-}1) \in \mathbb{R}^2$; so $\|\mathbf{v}\|_3 =$ ....

__ __ d) Our space is $\mathbb{R}$ with the usual Euclidean metric $\mathsf{d}(x, z) := |x - z|$.

I) These *closed* bnded non-void intervals $A_n :=$ ...., when unioned, form a set $\bigcup_{n=1}^{\infty} A_n =$ .... which is not closed.

__ __ II) Suppose that $U, V_1, V_2, \ldots$ are open sets of $\mathbb{R}$, and $E, K_1, K_2, \ldots$ are closed sets. **Circle** those of the following sets which are guaranteed to be *closed* in $\mathbb{R}$.

$E \smallsetminus U$. $U \smallsetminus E$. $K_1 \smallsetminus E$. $\bigcap_{n=1}^{\infty} K_n$.

$\mathbb{R} \smallsetminus [\bigcup_{n=1}^{\infty} V_n]$. $E \cup K_1$. $E \cap K_1$.

Essay questions: Please write each essay triple–spaced. Each essay starts a new page.

A2: In MS (Ω, d), sequence $\vec{b} \subset \Omega$ converges to both q and r in Ω. Prove that $q = r$, by showing that $\mathsf{d}(q, r) = 0$. [*Hint:* Use the Triangle Inequality.]

A3: In $\mathbb{R}$: Prove, for all sets $E_1, E_2 \subset \mathbb{R}$, that

1: $$\partial(E_1) \cup \partial(E_2) \quad \supset \quad \partial(E_1 \cap E_2)\,.$$

[*Hint:* Fixing a point $q \in \partial(E_1 \cap E_2)$, we know there exist sequences $\vec{\mathbf{b}} \subset E_1 \cap E_2$ and $\vec{\mathbf{x}} \subset [E_1 \cap E_2]^c$ converging to q. You need to show, either for j=1 or j=2, that ${E_j}^c$ includes a sequence $\vec{\mathbf{y}}$ that converges to q. Also, explain *why* the existance of such a $\vec{\mathbf{y}}$ is sufficient to establish (**??**).]

A1:	__ __	95pts
A2:	__ __	45pts
A3:	__ __	45pts
Total:	__ __ __	185pts

Please PRINT your name and ordinal. Ta:

.... Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....