Linear Algebra
MAS4105 *6137*

Class-A

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29Sep2015

Hello. All VSes here, $\mathbf{V}, \mathbf{H}, \mathbf{X}, \mathbf{Y}$, are $\mathbb{R}$-VSes. Use L_M for the lefthand trn defined by matrix M.

A1: Show no work. Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

___ ___ a) The 3×3 elem-matrix whose lefthand action adds 8 times row-2 to row-1 is $\begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$.

___ ___ b) In $\mathbb{R}^3$, let $\mathbf{u} := (8, 5, -1)$, $\mathbf{v} := (1, 0, -4)$ and $\mathbf{w} := (13, 10, 10)$. Circle: Then $\mathbf{w} \in \text{Line}(\mathbf{u}, \mathbf{v})$: *T* *F*

Then $\mathbf{w} \in \text{Spn}(\mathbf{u}, \mathbf{v})$: *T* *F*

Then $\text{Spn}(\mathbf{u}, \mathbf{v}, \mathbf{w}) = \mathbb{R}^3$: *T* *F*

___ ___ c) In each blank below, write either "there exist" or "for all", Circle one of the underlined scalar-pairs, and Circle a phrase.

Assertion $\boxed{\text{Spn}(\mathbf{v}, \mathbf{w}) \supset \text{Spn}(\mathbf{x}, \mathbf{y})}$ *means:*

"................... *scalars* <u>a, b</u> | <u>c, d</u> (*st.* | *we have that* | *and*)
.... *scalars* <u>a, b</u> | <u>c, d</u> (*st.* | *we have that*)
$\boxed{a\mathbf{v} + b\mathbf{w} = c\mathbf{x} + d\mathbf{y}}$."

___ ___ d) Let $\mathsf{B} := \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 3 & 6 & 0 & -3 & 0 \end{bmatrix}$. Then $\mathsf{R} := \mathit{RREF}(\mathsf{B})$ is [show no work, here]

$\mathsf{R} = \begin{bmatrix} \; & \; & \; & \; & \; \end{bmatrix}$.

I) For subspace $\mathbf{V} := \text{Nul}(\mathsf{L}_\mathsf{B})$, use back-substitution, and *scaling*, to produce an <u>*integer basis*</u>

$\mathbf{v}_1 := (\_, \_, \_, \_, \_)$ $\mathbf{v}_2 := (\_, \_, \_, \_, \_)$
$\mathbf{v}_3 := (\_, \_, \_, \_, \_)$. $\mathbf{v}_4 := (\_, \_, \_, \_, \_)$

[*Note:* **Only use as many as the dimension of V.**]

___ ___ e) The map $\text{PLY}_3 \to \text{PLY}_3$ which sends $f \mapsto g$, where $g(x) := x \cdot f'(x + 5)$, is: Circle best   Linear   Affine   Neither

f) Consider these two matrices:

$$\mathsf{R} := \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \quad \text{and} \quad \mathsf{A} := \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}.$$

Product matrix

$$[RA]^{40} = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}.$$

[*Hint:* You don't need multiply matrices. Geometrically, what motion do these matrices represent?]

A2: OYOP: Essay: *Write on every third line, so that I can easily write between the lines. In grammatical English sentences, prove the following:*

i) For linear map $\mathsf{T}: \mathbf{H} \to \mathbf{V}$ from VS $\mathbf{H}$ to $\mathbf{V}$, both finite-dimensional, *carefully* state the Rank–Nullity Thm.

ii) Give a careful proof, starting your argument with *"Proof of Rank–Nullity Thm"* and ending with *"QED"*.

[In your proof, use 0 for the <u>scalar</u> zero. In contrast, use $\vec{\mathbf{0}}_\mathbf{H}$ and $\vec{\mathbf{0}}_\mathbf{V}$ for the zero-vector in the two spaces.]

End of Class-A

A1:	___ ___ ___	130pts
A2:	___ ___	65pts
Total:	___ ___ ___	195pts