



NT-Cryptography  
MAT4930 7554

Class-A

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2Sep2015

A3: \_\_\_\_\_ 70pts

A4: \_\_\_\_\_ 95pts

Poorly stapled,  
or not double-spaced: \_\_\_\_\_ -15pts

Please *fill-in* every *blank* on this sheet.

**A3:** Show no work. Please write DNE in a blank if the described object does not exist or if the indicated operation cannot be performed.

Total: \_\_\_\_\_ 165pts

Please PRINT your name and ordinal. Ta:

Ord: \_\_\_\_\_

**a** Consider  $M := p \cdot q = 8549$ , where  $p < q$  are primes. Your mole in King's organization finds out that  $F := \varphi(M) = 8364$ .

Then  $p =$  \_\_\_\_\_  $< q =$  \_\_\_\_\_.

**b** Note  $p := 137$  is prime. The (multiplicative) order of 2 mod 137 is \_\_\_\_\_.

[Hint:  $p - 1$  has very few prime factors.]

**c** Note that  $\text{Gcd}(15, 21, 35) = 1$ . Find particular integers  $S, T, U$  so that  $15S + 21T + 35U = 1$ :

$S =$  \_\_\_\_\_,  $T =$  \_\_\_\_\_,  $U =$  \_\_\_\_\_.

[Hint:  $\text{Gcd}(\text{Gcd}(15, 21), 35) = 1$ .]

OYOP: In grammatical English *sentences*, write your essay on every *third* line (usually), so that I can easily write between the lines. Do not restate the question.

**A4:** Abstractly describe the RSA algorithm, what is public, what private, how to compute the decryption key from the encryption key, and how to encrypt and decrypt. (Do not bother to describe LBolt, nor Repeated-squaring, but do say where they are used.)

Alice's RSA code has modulus is  $M = 851$ , and encryption exponent  $\mathbf{E} := 317$ , both public. Bob has a message that can be interpreted as a number  $\beta$  in  $[0..M)$ . Since Alice knows the secret factorization  $M = p \cdot q$  into primes,  $p=37$ ,  $q=23$ , she can compute the decryption exponent  $\mathbf{d} =$  \_\_\_\_\_  $\in \mathbb{Z}_+$ . Bob's encrypted message  $\mu := \langle \beta^{\mathbf{E}} \rangle_M = 007$ . Alice decrypts it to  $\langle \mu^{\mathbf{d}} \rangle_M =$  \_\_\_\_\_  $\in [0..M)$ .

End of Class-A