Staple!

Ord: ___ ___

Sets and Logic
MHF3202 17HE

Class-A

Prof. JLF King
Wednesday, 16Feb2022

A5: Short answer. Show no work.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

___ ___ **a** Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$500 total] is a *really good idea*. A ten-page essay is fine. Date for the emailed-PDF is Sunday, March 27, 2022.

Circle: **Yes** **True** ***Résumé material!***

___ ___ **b** Note that $\mathrm{GCD}(15,21,35)=1$. Find particular integers S,T,U so that $15S+21T+35U=1$:

$S=$ ____________, $T=$ ____________, $U=$ ____________.

[*Hint:* $\mathrm{GCD}\big(\mathrm{GCD}(15,21),35\big)=1$.]

___ ___ **c** Write $N := 36300$ as a PoPP [Product of Prime Powers], $N \overset{\text{PoPP}}{=}$ ____________________. Hence the number of (positive) divisors of N is $\tau(N) =$ ________________.

___ ___ **d** The physics lab has atomic *zinc*, *tin*, *silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] ____________________ many possibilities.

This number *also* equals the number-of-ways of picking N candies from T many types of candy, where $N=$ __________ $\notin \{1,6\}$ and $T=$ __________ $\notin \{1,4\}$.

___ ___ **e** Mimicking what we did in class: From the 987×200 gameboard, cut-out (remove) the $(35,150)$-cell and one other cell at $P=(x,y)$. Circle those choices for P,

$(150,160)$, $(14,35)$, $(66,77)$, $(195,15)$, $(123,4)$

which, if removed, would leave a board that *definitely* can**not** be domino-tiled.

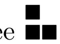
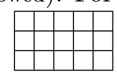
___ ___ **f** Both $\sim$ and $\bowtie$ are equiv-relations on a set Ω. Define binrels $\mathbf{I}$ and $\mathbf{U}$ on Ω as follows.

Define $\omega\,\mathbf{U}\,\lambda$ IFF *Either* $\omega \sim \lambda$ *or* $\omega \bowtie \lambda$ [or both].
Define $\omega\,\mathbf{I}\,\lambda$ IFF *Both* $\omega \sim \lambda$ *and* $\omega \bowtie \lambda$.

So "$\mathbf{U}$ *is an equiv-relation*" is: T F

So "$\mathbf{I}$ *is an equiv-relation*" is: T F

OYOP: *In grammatical English* ***sentences****, write your essays on every 2nd line (usually), so I can easily write between the lines.*

A6: An ***Lmino*** (pron. "ell-mino") comprises three  squares in an "L" shape (all four orientations are allowed). For natnum N, let $\mathbf{R}_N$ denote the $3{\times}N$ board: I.e,  is the $\mathbf{R}_5$ board. Prove:

Theorem: When N is odd, then board $\mathbf{R}_N$ is not Lmino-tilable.

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, *labeled* pictures.

When N is even, our $\mathbf{R}_N$ has exactly __________ *many Lmino-tilings.*

A5:	___ ___ ___	115pts
A6:	___ ___	55pts
Total:	___ ___ ___	170pts

NAME: ____

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ____