Staple!

Sets and Logic
MHF3202 12A3

Class-A

Prof. JLF King
22 Oct. 2021

A4: Short answer. Show no work.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

___ ___ a) Prof. King thinks that submitting a ROBERT LONG PRIZE ESSAY [typically 2 prizes, \$500 total] is a *really good idea.* A ten-page essay is fine. Date for the emailed-PDF is Sunday, March 27, 2022.

Circle: **Yes** **True** ***Résumé material!***

___ ___ b) In $[5x^2 + 4y + z^3 + 7]^{20}$, compute these coeffs:

$\text{Coeff}(x^6 z^8) =$ ________________ .

$\text{Coeff}(y^5 z^6) =$ ________________

[You may write answers as a product numbers, powers and multinomial-coeffs.]

___ ___ c) Write $N := 36300$ as a PoPP [Product of Prime Powers], $N \stackrel{\text{PoPP}}{=}$ ________________ . Hence the number of (positive) divisors of N is $\tau(N) =$ ________________ .

___ ___ d) Suppose that $\prec$ is a total-order on set $\mathcal{S}$, and $<$ is total-order on set $\mathbf{\Omega}$, both strict. Define binrel $\ll$ on $\mathcal{S} \times \mathbf{\Omega}$ by:

$$(b, \beta) \ll (c, \gamma)$$

IFF *Either* $b \prec c$ *or* $[b = c$ and $\beta < \gamma]$.

Then:
Relation $\ll$ is a total-order. T F

Suppose $\prec$ and $<$ are each well-orders.

Then $\ll$ is a well-order. T F

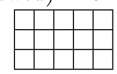
___ ___ e) Mimicking what we did in class: From the 987×200 game-board, cut-out (remove) the $(35, 150)$-cell and one other cell at $P = (x, y)$. Circle those choices for P,

$(150, 160)$, $(14, 35)$, $(66, 77)$, $(195, 15)$, $(123, 4)$

which, if removed, would leave a board that *definitely* can**not** be domino-tiled.

___ ___ f) Both $\sim$ and $\bowtie$ are equiv-relations on a set Ω. Define binrels $\mathbf{I}$ and $\mathbf{U}$ on Ω as follows.
Define $\omega \,\mathbf{U}\, \lambda$ IFF *Either* $\omega \sim \lambda$ *or* $\omega \bowtie \lambda$ [or both].
Define $\omega \,\mathbf{I}\, \lambda$ IFF *Both* $\omega \sim \lambda$ *and* $\omega \bowtie \lambda$.
So "$\mathbf{U}$ *is an equiv-relation*" is: T F
So "$\mathbf{I}$ *is an equiv-relation*" is: T F

OYOP: *In grammatical English sentences, write your essays on every 2nd line (usually), so I can easily write between the lines.*

A5: An ***Lmino*** (pron. "ell-mino") comprises three  squares in an "L" shape (all four orientations are allowed). For natnum N, let $\mathbf{R}_N$ denote the $3 \times N$ board: I.e,  is the $\mathbf{R}_5$ board. Prove:

Theorem: When N is odd, then board $\mathbf{R}_N$ is not Lmino-tilable.

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, *labeled* pictures.

When N is even, our $\mathbf{R}_N$ has exactly ________ *many Lmino-tilings.*

A4:	___ ___ ___	110pts
A5:	___ ___	55pts
Total:	___ ___ ___	165pts

NAME: ________________________________

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________________________________