

A3: Short answer. Show no work.

Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

True! Yes! wH'at S a?sEnTENcE

A3: _____ 70pts

A4: _____ 35pts

A5: _____ 35pts

Total: _____ 140pts

b $B^E = \sum_{j=0}^{59} \binom{59}{j} \cdot 4^{2j}$, for posints $B=$ _____ & $E=$ _____.

c The physics lab has atomic *zinc, tin, silver* and *gold*. I'm allowed to take 6 atoms, so I have [expressed as single integer] _____ many possibilities.

d There are $\binom{K}{J}$ many [diagonal] lattice-paths from point $(0, 2)$ to $(21, 7)$, where $K=$ _____ and $J=$ _____.

Such a path is **bad**, if it touches the x -axis. And $|\text{BAD}| = \binom{N}{L}$, where $N=$ _____ and $L=$ _____.

OYOP: *In grammatical English sentences, write your essays on every **third** line (usually), so that I can easily write between the lines.* Start each essay on a new sheet-of-paper. Please number the pages "1 of 57", "2 of 57"... (or "1/57", "2/57"...). I suggest you put your name on each sheet.

A4: Give a careful bijective proof of:

Thm: *Fix a natnum $N \geq 3$. Then*

$$* : \quad \llbracket N \downarrow 3 \rrbracket \cdot 2^{N-3} = \sum_{k=3}^N \llbracket k \downarrow 3 \rrbracket \cdot \binom{N}{k}.$$

A5: Prove, for each natnum N , that

$$\sum_{k=0}^N \left[\binom{N}{k}^2 \right] = \binom{2N}{N}.$$