

(This is much longer than the actual exam.)

OYOP: In grammatical English *sentences*, write your essay on every *third* line (usually), so that I can easily write between the lines. Do **not** restate the question.

For $n, k \in \mathbb{Z}$, let $n \perp k$ mean $\text{Gcd}(n, k) = 1$. Let **E.G** abbreviate *Euclidean Geometry*.

A1: Integers a, b, c form a Pythagorean triple, $a^2 + b^2 = c^2$. Suppose $a \not\perp b$. Prove that $b \not\perp c$.

A2: Show by explicit example that SAS does not hold in the taxicab geometry on \mathbb{R}^2 .

A3: In E.G, prove that the three angle-bisectors of $\triangle ABC$ intersect at a point; call it P . What is the name of this point?

A4: Carefully state the *Central-angle theorem* for a circle. Prove the *Central-angle thm*.

A5: State and prove the *Pythagorean theorem*.

A6: On a set Y , a *metric* m is a map $Y \times Y \rightarrow \mathbb{R}$ such that $\forall x, y, z \in Y$:

MS1: $m(x, y) \geq 0$ and $m(x, y) = 0$ iff $x = y$.

MS2: $m(x, y) = m(y, x)$.

MS3: $m(x, z) \leq m(x, y) + m(y, z)$.

MS4: $m(x, y) \leq m(x, z) + m(z, y)$.

A7: On \mathbb{R} -VS X , a *norm* $\|\cdot\|$ is a map $X \rightarrow \mathbb{R}$ satisfying these three axioms. [Hint: Quantifiers.]

N1: $\|x\| \geq 0$ and $\|x\| = 0$ iff $x = 0$.

N2: $\|ax\| = |a| \|x\|$.

N3: $\|x + y\| \leq \|x\| + \|y\|$.

N4: $\|x - y\| \leq \|x\| + \|y\|$.

A8: Short answer. Show no work.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

a Vertices $A := (8, 4), B := (-6, -2), C := (-2, -2)$ form a triangle T whose circum-center is (\quad, \quad) .

Also, Centroid(T) = (\quad, \quad) .

b A particular Pythagorean triple has $a^2 + 40^2 = c^2$, where $a = \quad$ and $c = \quad$.

c Let $\mathbf{v} := (3, -3, -1, 1, 2) \in \mathbb{R}^5$; so $\|\mathbf{v}\|_3 = \quad$.

End of Prac-A

Please PRINT your name and ordinal. Ta:

Ord:

HONOR CODE: "I have neither requested nor received help on this exam other than from my professor."

Signature: