Useful notation for exam

F is a field, and **u**,**v**,**w** $\in$ F. $\text{MAT}_{3\times5}(\mathsf{F})$ is the set of 3×5-matrices in F, written $\text{MAT}(\mathsf{F})$ if the 3×5 is understood or $\text{MAT}_{3\times5}$ if the field is understood, or just MAT if both are understood.

For $\mathsf{A} \in \text{MAT}_{3\times5}$, let L_A be the ***lefthand action*** of A; the linear trn mapping F^5 to F^3, sending column-vectors to column-vectors.

For p an odd prime, always give answers in *symmetric residues*, e.g working over $\mathbb{Z}_{17}$, answers should be in $[-8\,..\,8]$. The phrase "*working over*" a field F means that all vectors, matrices, LinCombs etc., are over that field. If the field is $\mathbb{Z}_p$, then $\equiv$ means $\equiv_p$. E.g: "*Working over* $\mathbb{Z}_{19}$, *compute the determinant and inverse of* $\mathsf{M} := \begin{bmatrix} 21 & 15 \\ 10 & 4 \end{bmatrix}$."

Soln. Reducing, $\mathsf{M} \equiv \begin{bmatrix} 2 & -4 \\ -9 & 4 \end{bmatrix}$, so $D := \text{Det}(\mathsf{M})$ equals

$$2 \cdot 4 - [-4][-9] = 8 - 36 \equiv 8 + 2 = 10 \equiv -9 .$$

Hence reciprocal $R := \langle 1 \div D \rangle_{19} = 2$. Thus

$$\mathsf{M}^{-1} \equiv R \cdot \begin{bmatrix} 4 & 4 \\ 9 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 18 & 4 \end{bmatrix} \equiv \begin{bmatrix} 8 & 8 \\ -1 & 4 \end{bmatrix}.$$

Checking

$$\begin{bmatrix} 2 & -4 \\ -9 & 4 \end{bmatrix} \cdot \begin{bmatrix} 8 & 8 \\ -1 & 4 \end{bmatrix} \overset{\text{expect}}{\equiv} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$ ♦

"*Working over* $\mathbb{C}$, *compute the determinant of* $\mathsf{B} := \begin{bmatrix} 2+i & 3 \\ 1 & 1+i \end{bmatrix}$. Compute s, the $(1,2)$-entry of B^{-1}."

Well, $\delta := \text{Det}(\mathsf{B}) = -2 + 3\mathbf{i}$. Thus $\frac{1}{\delta} = \overline{\delta}/|\delta|^2 = \overline{\delta}/13$. So $13s$ equals $\overline{\delta} \cdot [-3] = 6 + 9\mathbf{i}$. So $s = \frac{6}{13} + \frac{9}{13}\mathbf{i}$. ♦

§A Reciprocal tables in $\mathbb{Z}_p$

Modulo 2:

x	$\langle 1/x \rangle_2$
1	1

Modulo 3:

x	$\langle 1/x \rangle_3$
±1	±1

Modulo 5:

x	$\langle 1/x \rangle_5$	x	$\langle 1/x \rangle_5$
±1	±1	±2	∓2

Modulo 7:

x	$\langle 1/x \rangle_7$	x	$\langle 1/x \rangle_7$
±1	±1		
±2	∓3	±3	∓2

Modulo 11:

x	$\langle 1/x \rangle_{11}$	x	$\langle 1/x \rangle_{11}$
±1	±1		
±2	∓5	±4	±3
±3	±4	±5	∓2

Modulo 13:

x	$\langle 1/x \rangle_{13}$	x	$\langle 1/x \rangle_{13}$
±1	±1	±4	∓3
±2	∓6	±5	∓5
±3	∓4	±6	∓2

Modulo 17:

x	$\langle 1/x \rangle_{17}$	x	$\langle 1/x \rangle_{17}$
±1	±1	±5	±7
±2	∓8	±6	±3
±3	±6	±7	±5
±4	∓4	±8	∓8

Modulo 19:

x	$\langle 1/x \rangle_{19}$	x	$\langle 1/x \rangle_{19}$
±1	±1		
±2	∓9	±6	∓3
±3	∓6	±7	∓8
±4	±5	±8	∓7
±5	±4	±9	∓2